Maple 2018.2 Integration Test Results
on the problems in "6 Hyperbolic functions/6.3 Hyperbolic tangent"

Test results for the 23 problems in "6.3.1 (c+d x)^m (a+b tanh)^n.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int (d x+c)^3 \tanh(f x+e)\, dx$$

Optimal(type 4, 109 leaves, 6 steps):

$$-\frac{(d x+c)^4}{4 d}+\frac{(d x+c)^3 \ln\left(1+e^{2 f x+2 e}\right)}{f}+\frac{3 d (d x+c)^2 \operatorname{polylog}\left(2,-e^{2 f x+2 e}\right)}{2 f^2}-\frac{3 d^2 (d x+c) \operatorname{polylog}\left(3,-e^{2 f x+2 e}\right)}{2 f^3}+\frac{3 d^3 \operatorname{polylog}\left(4,-e^{2 f x+2 e}\right)}{4 f^4}$$

Result(type 4, 393 leaves):

$$-c d^2 x^3-\frac{2 c^3 \ln\left(e^{f x+e}\right)}{f}+\frac{c^3 \ln\left(1+e^{2 f x+2 e}\right)}{f}-\frac{3 d^3 e^4}{2 f^4}+\frac{3 c d^2 \operatorname{polylog}\left(2,-e^{2 f x+2 e}\right) x}{f^2}+\frac{3 c^2 d \ln\left(1+e^{2 f x+2 e}\right) x}{f}+\frac{6 d e c^2 \ln\left(e^{f x+e}\right)}{f^2}$$
$$-\frac{6 d^2 e^2 c \ln\left(e^{f x+e}\right)}{f^3}+\frac{6 c d^2 e^2 x}{f^2}-\frac{6 c^2 d e x}{f}+\frac{3 c d^2 \ln\left(1+e^{2 f x+2 e}\right) x^2}{f}+c^3 x-\frac{d^3 x^4}{4}-\frac{2 d^3 e^3 x}{f^3}+\frac{4 c d^2 e^3}{f^3}-\frac{3 c^2 d e^2}{f^2}+\frac{d^3 \ln\left(1+e^{2 f x+2 e}\right) x^3}{f}$$
$$+\frac{3 d^3 \operatorname{polylog}\left(2,-e^{2 f x+2 e}\right) x^2}{2 f^2}-\frac{3 d^3 \operatorname{polylog}\left(3,-e^{2 f x+2 e}\right) x}{2 f^3}-\frac{3 c d^2 \operatorname{polylog}\left(3,-e^{2 f x+2 e}\right)}{2 f^3}+\frac{3 c^2 d \operatorname{polylog}\left(2,-e^{2 f x+2 e}\right)}{2 f^2}+\frac{2 d^3 e^3 \ln\left(e^{f x+e}\right)}{f^4}$$
$$-\frac{3 c^2 d x^2}{2}+\frac{3 d^3 \operatorname{polylog}\left(4,-e^{2 f x+2 e}\right)}{4 f^4}$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int (d x+c) \tanh(f x+e)\, dx$$

Optimal(type 4, 53 leaves, 4 steps):

$$-\frac{(d x+c)^2}{2 d}+\frac{(d x+c) \ln\left(1+e^{2 f x+2 e}\right)}{f}+\frac{d \operatorname{polylog}\left(2,-e^{2 f x+2 e}\right)}{2 f^2}$$

Result(type 4, 108 leaves):

$$-\frac{d x^2}{2}+c x+\frac{c \ln\left(1+e^{2 f x+2 e}\right)}{f}-\frac{2 c \ln\left(e^{f x+e}\right)}{f}-\frac{2 d e x}{f}-\frac{d e^2}{f^2}+\frac{d \ln\left(1+e^{2 f x+2 e}\right) x}{f}+\frac{d \operatorname{polylog}\left(2,-e^{2 f x+2 e}\right)}{2 f^2}+\frac{2 d e \ln\left(e^{f x+e}\right)}{f^2}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int (d x+c)^2 \tanh(f x+e)^2\, dx$$

Optimal(type 4, 86 leaves, 6 steps):

$$-\frac{(d x+c)^2}{f}+\frac{(d x+c)^3}{3 d}+\frac{2 d (d x+c) \ln\left(1+e^{2 f x+2 e}\right)}{f^2}+\frac{d^2 \operatorname{polylog}\left(2,-e^{2 f x+2 e}\right)}{f^3}-\frac{(d x+c)^2 \tanh(f x+e)}{f}$$

Result(type 4, 176 leaves):

$$\frac{d^2 x^3}{3}+c d x^2+c^2 x+\frac{2 (d^2 x^2+2 c d x+c^2)}{f(1+e^{2 f x+2 e})}+\frac{2 d c \ln(1+e^{2 f x+2 e})}{f^2}-\frac{4 d c \ln(e^{f x+e})}{f^2}-\frac{2 d^2 x^2}{f}-\frac{4 d^2 e x}{f^2}-\frac{2 d^2 e^2}{f^3}+\frac{2 d^2 \ln(1+e^{2 f x+2 e}) x}{f^2}$$
$$+\frac{d^2 \operatorname{polylog}(2, -e^{2 f x+2 e})}{f^3}+\frac{4 d^2 e \ln(e^{f x+e})}{f^3}$$

Problem 6: Unable to integrate problem.

$$\int (d x+c) (b \tanh(f x+e))^{5/2} dx$$

Optimal(type 4, 1108 leaves, 44 steps):

$$\frac{2 b^{5/2} d \arctan\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{b}}\right)}{3 f^2}-\frac{(-b)^{5/2} (d x+c) \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{-b}}\right)}{f}-\frac{(-b)^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{-b}}\right)^2}{2 f^2}$$
$$+\frac{2 b^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{b}}\right)}{3 f^2}+\frac{b^{5/2} (d x+c) \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{b}}\right)}{f}+\frac{b^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{b}}\right)^2}{2 f^2}$$
$$-\frac{b^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{b}}\right) \ln\left(\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \tanh(f x+e)}}\right)}{f^2}+\frac{b^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{b}}\right) \ln\left(\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \tanh(f x+e)}}\right)}{f^2}$$
$$-\frac{b^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{b}}\right) \ln\left(\frac{2 \sqrt{b} \left(\sqrt{-b}-\sqrt{b \tanh(f x+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right) \left(\sqrt{b}+\sqrt{b \tanh(f x+e)}\right)}\right)}{2 f^2}$$
$$-\frac{b^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{b}}\right) \ln\left(\frac{2 \sqrt{b} \left(\sqrt{-b}+\sqrt{b \tanh(f x+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right) \left(\sqrt{b}+\sqrt{b \tanh(f x+e)}\right)}\right)}{2 f^2}$$
$$+\frac{(-b)^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{-b}}\right) \ln\left(\frac{2}{1-\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{-b}}}\right)}{f^2}$$
$$-\frac{(-b)^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{-b}}\right) \ln\left(\frac{2 \left(\sqrt{b}-\sqrt{b \tanh(f x+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right) \left(1-\frac{\sqrt{b \tanh(f x+e)}}{\sqrt{-b}}\right)}\right)}{2 f^2}$$

$$-\frac{(-b)^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(-\frac{2\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{2f^2}$$

$$-\frac{(-b)^{5/2} d \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2}{1+\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{f^2}-\frac{b^{5/2} d \operatorname{polylog}\left(2, 1-\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(fx+e)}}\right)}{2f^2}$$

$$-\frac{b^{5/2} d \operatorname{polylog}\left(2, 1-\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(fx+e)}}\right)}{2f^2}+\frac{b^{5/2} d \operatorname{polylog}\left(2, 1-\frac{2\sqrt{b}\left(\sqrt{-b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)}{4f^2}$$

$$+\frac{b^{5/2} d \operatorname{polylog}\left(2, 1-\frac{2\sqrt{b}\left(\sqrt{-b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)}{4f^2}+\frac{(-b)^{5/2} d \operatorname{polylog}\left(2, 1-\frac{2}{1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{2f^2}$$

$$-\frac{(-b)^{5/2} d \operatorname{polylog}\left(2, 1-\frac{2\left(\sqrt{b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{4f^2}-\frac{(-b)^{5/2} d \operatorname{polylog}\left(2, 1+\frac{2\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{4f^2}$$

$$+\frac{(-b)^{5/2} d \operatorname{polylog}\left(2, 1-\frac{2}{1+\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{2f^2}-\frac{4 b^2 d\sqrt{b\tanh(fx+e)}}{3f^2}-\frac{2b(dx+c)(b\tanh(fx+e))^{3/2}}{3f}$$

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Result(type 8, 18 leaves):
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$$\int (dx+c)(b\tanh(fx+e))^{5/2}\,dx$$

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Problem 7: Unable to integrate problem.
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$$\int (dx+c)(b\tanh(fx+e))^{3/2}\,dx$$

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Optimal(type 4, 1089 leaves, 43 steps):
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$$-\frac{2b^{3/2} d \arctan\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)}{f^2}-\frac{(-b)^{3/2}(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}{f}-\frac{(-b)^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)^2}{2f^2}$$

$$+\frac{2 b^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{b}}\right)}{f^2}+\frac{b^{3/2}(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{b}}\right)}{f}+\frac{b^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{b}}\right)^2}{2 f^2}$$

$$-\frac{b^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b \tanh(fx+e)}}\right)}{f^2}+\frac{b^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b \tanh(fx+e)}}\right)}{f^2}$$

$$-\frac{b^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}\left(\sqrt{-b}-\sqrt{b \tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b \tanh(fx+e)}\right)}\right)}{2 f^2}$$

$$-\frac{b^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}\left(\sqrt{-b}+\sqrt{b \tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b \tanh(fx+e)}\right)}\right)}{2 f^2}$$

$$+\frac{(-b)^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2}{1-\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{-b}}}\right)}{f^2}$$

$$-\frac{(-b)^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2\left(\sqrt{b}-\sqrt{b \tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{2 f^2}$$

$$-\frac{(-b)^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(-\frac{2\left(\sqrt{b}+\sqrt{b \tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{2 f^2}$$

$$-\frac{(-b)^{3/2} d \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2}{1+\frac{\sqrt{b \tanh(fx+e)}}{\sqrt{-b}}}\right)}{f^2}-\frac{b^{3/2} d \operatorname{polylog}\left(2, 1-\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b \tanh(fx+e)}}\right)}{2 f^2}$$

$$-\frac{b^{3/2} d \operatorname{polylog}\left(2, 1-\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b \tanh(fx+e)}}\right)}{2 f^2}+\frac{b^{3/2} d \operatorname{polylog}\left(2, 1-\frac{2\sqrt{b}\left(\sqrt{-b}-\sqrt{b \tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b \tanh(fx+e)}\right)}\right)}{4 f^2}$$

$$+\frac{b^{3/2} d \operatorname{polylog}\left(2, 1-\frac{2\sqrt{b}\left(\sqrt{-b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)}{4 f^2}+\frac{(-b)^{3/2} d \operatorname{polylog}\left(2, 1-\frac{2}{1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{2 f^2}$$

$$-\frac{(-b)^{3/2} d \operatorname{polylog}\left(2, 1-\frac{2\left(\sqrt{b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{4 f^2}-\frac{(-b)^{3/2} d \operatorname{polylog}\left(2, 1+\frac{2\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{4 f^2}$$

$$+\frac{(-b)^{3/2} d \operatorname{polylog}\left(2, 1-\frac{2}{1+\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{2 f^2}-\frac{2 b (d x+c) \sqrt{b\tanh(fx+e)}}{f}$$

Result(type 8, 18 leaves):

$$\int (d x+c) (b\tanh(fx+e))^{3/2}\, dx$$

Problem 8: Unable to integrate problem.

$$\int (d x+c) \sqrt{b\tanh(fx+e)}\, dx$$

Optimal(type 4, 1020 leaves, 37 steps):

$$-\frac{(d x+c) \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\sqrt{-b}}{f}-\frac{d \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)^2\sqrt{-b}}{2 f^2}+\frac{d \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2}{1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)\sqrt{-b}}{f^2}$$

$$-\frac{d \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2\left(\sqrt{b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)\sqrt{-b}}{2 f^2}$$

$$-\frac{d \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(-\frac{2\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)\sqrt{-b}}{2 f^2}$$

$$-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2}{1+\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)\sqrt{-b}}{f^2}+\frac{d\operatorname{polylog}\left(2,1-\frac{2}{1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)\sqrt{-b}}{2f^2}$$

$$-\frac{d\operatorname{polylog}\left(2,1-\frac{2\left(\sqrt{b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)\sqrt{-b}}{4f^2}-\frac{d\operatorname{polylog}\left(2,1+\frac{2\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)\sqrt{-b}}{4f^2}$$

$$+\frac{d\operatorname{polylog}\left(2,1-\frac{2}{1+\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)\sqrt{-b}}{2f^2}+\frac{(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\sqrt{b}}{f}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)^2\sqrt{b}}{2f^2}$$

$$-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(fx+e)}}\right)\sqrt{b}}{f^2}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(fx+e)}}\right)\sqrt{b}}{f^2}$$

$$-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}\left(\sqrt{-b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)\sqrt{b}}{2f^2}$$

$$-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}\left(\sqrt{-b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)\sqrt{b}}{2f^2}-\frac{d\operatorname{polylog}\left(2,1-\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(fx+e)}}\right)\sqrt{b}}{2f^2}$$

$$-\frac{d\operatorname{polylog}\left(2,1-\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(fx+e)}}\right)\sqrt{b}}{2f^2}+\frac{d\operatorname{polylog}\left(2,1-\frac{2\sqrt{b}\left(\sqrt{-b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)\sqrt{b}}{4f^2}$$

$$+\frac{d\operatorname{polylog}\left(2,1-\frac{2\sqrt{b}\left(\sqrt{-b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)\sqrt{b}}{4f^2}$$

Result(type 8, 18 leaves):

$$\int (dx+c)\sqrt{b\tanh(fx+e)}\,dx$$

Problem 9: Unable to integrate problem.

$$\int \frac{dx+c}{\sqrt{b\tanh(fx+e)}}\,dx$$

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Optimal(type 4, 1020 leaves, 37 steps):
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$$-\frac{(d x+c)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}{f\sqrt{-b}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)^2}{2f^2\sqrt{-b}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2}{1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{f^2\sqrt{-b}}$$

$$-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2\left(\sqrt{b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{2f^2\sqrt{-b}}$$

$$-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(-\frac{2\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{2f^2\sqrt{-b}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)\ln\left(\frac{2}{1+\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{f^2\sqrt{-b}}$$

$$+\frac{d\operatorname{polylog}\left(2,1-\frac{2}{1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{2f^2\sqrt{-b}}-\frac{d\operatorname{polylog}\left(2,1-\frac{2\left(\sqrt{b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{4f^2\sqrt{-b}}$$

$$-\frac{d\operatorname{polylog}\left(2,1+\frac{2\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(1-\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}\right)}\right)}{4f^2\sqrt{-b}}+\frac{d\operatorname{polylog}\left(2,1-\frac{2}{1+\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{-b}}}\right)}{2f^2\sqrt{-b}}$$

$$+\frac{(d x+c)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)}{f\sqrt{b}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)^2}{2f^2\sqrt{b}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(fx+e)}}\right)}{f^2\sqrt{b}}$$

$$+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(fx+e)}}\right)}{f^2\sqrt{b}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}\left(\sqrt{-b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)}{2f^2\sqrt{b}}$$

$$-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(fx+e)}}{\sqrt{b}}\right)\ln\left(\frac{2\sqrt{b}\left(\sqrt{-b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)}{2f^2\sqrt{b}}-\frac{d\operatorname{polylog}\left(2,1-\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(fx+e)}}\right)}{2f^2\sqrt{b}}$$

$$-\frac{d\,\text{polylog}\left(2, 1-\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(fx+e)}}\right)}{2f^2\sqrt{b}}+\frac{d\,\text{polylog}\left(2, 1-\frac{2\sqrt{b}\left(\sqrt{-b}-\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}-\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)}{4f^2\sqrt{b}}$$
$$+\frac{d\,\text{polylog}\left(2, 1-\frac{2\sqrt{b}\left(\sqrt{-b}+\sqrt{b\tanh(fx+e)}\right)}{\left(\sqrt{-b}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{b\tanh(fx+e)}\right)}\right)}{4f^2\sqrt{b}}$$

Result(type 8, 18 leaves):

$$\int \frac{dx+c}{\sqrt{b\tanh(fx+e)}}\,dx$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^2 (a+b\tanh(fx+e))^2\,dx$$

Optimal(type 4, 205 leaves, 13 steps):

$$-\frac{b^2(dx+c)^2}{f}+\frac{a^2(dx+c)^3}{3d}-\frac{2ab(dx+c)^3}{3d}+\frac{b^2(dx+c)^3}{3d}+\frac{2b^2d(dx+c)\ln\left(1+e^{2fx+2e}\right)}{f^2}+\frac{2ab(dx+c)^2\ln\left(1+e^{2fx+2e}\right)}{f}$$
$$+\frac{b^2d^2\,\text{polylog}\left(2,-e^{2fx+2e}\right)}{f^3}+\frac{2abd(dx+c)\,\text{polylog}\left(2,-e^{2fx+2e}\right)}{f^2}-\frac{abd^2\,\text{polylog}\left(3,-e^{2fx+2e}\right)}{f^3}-\frac{b^2(dx+c)^2\tanh(fx+e)}{f}$$

Result(type 4, 509 leaves):

$$\frac{4bad^2e^2x}{f^2}-\frac{4bacde^2}{f^2}+\frac{2bad^2\ln\left(1+e^{2fx+2e}\right)x^2}{f}+\frac{2bad^2\,\text{polylog}\left(2,-e^{2fx+2e}\right)x}{f^2}-\frac{4bad^2e^2\ln\left(e^{fx+e}\right)}{f^3}-\frac{4b^2d^2ex}{f^2}+\frac{8bad^2e^3}{3f^3}$$
$$+\frac{2b^2d^2\ln\left(1+e^{2fx+2e}\right)x}{f^2}+\frac{4b^2d^2e\ln\left(e^{fx+e}\right)}{f^3}+\frac{2bac^2\ln\left(1+e^{2fx+2e}\right)}{f}-\frac{4bac^2\ln\left(e^{fx+e}\right)}{f}+\frac{2b^2cd\ln\left(1+e^{2fx+2e}\right)}{f^2}-\frac{4b^2cd\ln\left(e^{fx+e}\right)}{f^2}$$
$$-2abcdx^2-\frac{abd^2\,\text{polylog}\left(3,-e^{2fx+2e}\right)}{f^3}+a^2cdx^2+b^2cdx^2+\frac{b^2d^2\,\text{polylog}\left(2,-e^{2fx+2e}\right)}{f^3}+\frac{2bacd\,\text{polylog}\left(2,-e^{2fx+2e}\right)}{f^2}+\frac{a^2d^2x^3}{3}+\frac{b^2d^2x^3}{3}$$
$$+a^2c^2x+b^2c^2x-\frac{2b^2d^2x^2}{f}-\frac{2b^2d^2e^2}{f^3}+\frac{2b^2\left(d^2x^2+2cdx+c^2\right)}{f\left(1+e^{2fx+2e}\right)}-\frac{2abd^2x^3}{3}+2abc^2x-\frac{8bacdex}{f}+\frac{4b\ln\left(1+e^{2fx+2e}\right)acdx}{f}$$
$$+\frac{8bacde\ln\left(e^{fx+e}\right)}{f^2}$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^2 (a+b\tanh(fx+e))^3\,dx$$

Optimal(type 4, 393 leaves, 22 steps):

$$\frac{b^3 c d x}{f}+\frac{b^3 d^2 x^2}{2 f}-\frac{3 a b^2 (d x+c)^2}{f}+\frac{a^3 (d x+c)^3}{3 d}-\frac{a^2 b (d x+c)^3}{d}+\frac{a b^2 (d x+c)^3}{d}-\frac{b^3 (d x+c)^3}{3 d}+\frac{6 a b^2 d (d x+c) \ln(1+e^{2 f x+2 e})}{f^2}$$
$$+\frac{3 a^2 b (d x+c)^2 \ln(1+e^{2 f x+2 e})}{f}+\frac{b^3 (d x+c)^2 \ln(1+e^{2 f x+2 e})}{f}+\frac{b^3 d^2 \ln(\cosh(f x+e))}{f^3}+\frac{3 a b^2 d^2 \operatorname{polylog}(2,-e^{2 f x+2 e})}{f^3}$$
$$+\frac{3 a^2 b d (d x+c) \operatorname{polylog}(2,-e^{2 f x+2 e})}{f^2}+\frac{b^3 d (d x+c) \operatorname{polylog}(2,-e^{2 f x+2 e})}{f^2}-\frac{3 a^2 b d^2 \operatorname{polylog}(3,-e^{2 f x+2 e})}{2 f^3}-\frac{b^3 d^2 \operatorname{polylog}(3,-e^{2 f x+2 e})}{2 f^3}$$
$$-\frac{b^3 d (d x+c) \tanh(f x+e)}{f^2}-\frac{3 a b^2 (d x+c)^2 \tanh(f x+e)}{f}-\frac{b^3 (d x+c)^2 \tanh(f x+e)^2}{2 f}$$

Result(type 4, 1021 leaves):

$$\frac{4 b a^2 d^2 e^3}{f^3}+\frac{2 b^3 d^2 e^2 x}{f^2}-\frac{2 b^3 c d e^2}{f^2}-\frac{6 b^2 a d^2 x^2}{f}-\frac{6 b^2 a d^2 e^2}{f^3}+\frac{3 b a^2 c^2 \ln(1+e^{2 f x+2 e})}{f}-\frac{2 b^3 d^2 e^2 \ln(e^{f x+e})}{f^3}+\frac{b^3 \ln(1+e^{2 f x+2 e}) d^2 x^2}{f}$$
$$+\frac{b^3 \operatorname{polylog}(2,-e^{2 f x+2 e}) d^2 x}{f^2}+\frac{b^3 c d \operatorname{polylog}(2,-e^{2 f x+2 e})}{f^2}-\frac{6 b a^2 c^2 \ln(e^{f x+e})}{f}-3 a^2 b c d x^2+3 a b^2 c d x^2$$
$$+\frac{1}{f^2 (1+e^{2 f x+2 e})^2}\left(2 b^2 \left(3 a d^2 f x^2 e^{2 f x+2 e}+b d^2 f x^2 e^{2 f x+2 e}+6 a c d f x e^{2 f x+2 e}+2 b c d f x e^{2 f x+2 e}+3 a c^2 f e^{2 f x+2 e}+3 a d^2 f x^2+b c^2 f e^{2 f x+2 e}\right.\right.$$
$$\left.\left.+b d^2 x e^{2 f x+2 e}+6 a c d f x+b c d e^{2 f x+2 e}+3 a c^2 f+b d^2 x+b c d\right)\right)+3 a^2 b c^2 x+3 a b^2 c^2 x-a^2 b d^2 x^3+a b^2 d^2 x^3+a^3 c d x^2-b^3 c d x^2+\frac{4 b^3 d^2 e^3}{3 f^3}$$
$$+\frac{b^3 d^2 \ln(1+e^{2 f x+2 e})}{f^3}-\frac{2 b^3 d^2 \ln(e^{f x+e})}{f^3}+\frac{b^3 c^2 \ln(1+e^{2 f x+2 e})}{f}-\frac{2 b^3 c^2 \ln(e^{f x+e})}{f}+\frac{a^3 d^2 x^3}{3}-\frac{b^3 d^2 x^3}{3}+a^3 c^2 x+b^3 c^2 x-\frac{12 b a^2 c d e x}{f}$$
$$+\frac{6 b \ln(1+e^{2 f x+2 e}) a^2 c d x}{f}+\frac{12 b a^2 c d e \ln(e^{f x+e})}{f^2}+\frac{3 a b^2 d^2 \operatorname{polylog}(2,-e^{2 f x+2 e})}{f^3}-\frac{3 a^2 b d^2 \operatorname{polylog}(3,-e^{2 f x+2 e})}{2 f^3}$$
$$-\frac{b^3 d^2 \operatorname{polylog}(3,-e^{2 f x+2 e})}{2 f^3}-\frac{6 b a^2 c d e^2}{f^2}+\frac{6 b a^2 d^2 e^2 x}{f^2}-\frac{4 b^3 c d e x}{f}-\frac{12 b^2 a d^2 e x}{f^2}+\frac{3 b \ln(1+e^{2 f x+2 e}) a^2 d^2 x^2}{f}$$
$$+\frac{3 b \operatorname{polylog}(2,-e^{2 f x+2 e}) a^2 d^2 x}{f^2}+\frac{3 b a^2 c d \operatorname{polylog}(2,-e^{2 f x+2 e})}{f^2}+\frac{12 b^2 a d^2 e \ln(e^{f x+e})}{f^3}-\frac{6 b a^2 d^2 e^2 \ln(e^{f x+e})}{f^3}+\frac{6 b^2 \ln(1+e^{2 f x+2 e}) a d^2 x}{f^2}$$
$$+\frac{2 b^3 \ln(1+e^{2 f x+2 e}) c d x}{f}+\frac{4 b^3 c d e \ln(e^{f x+e})}{f^2}-\frac{12 b^2 a c d \ln(e^{f x+e})}{f^2}+\frac{6 b^2 a c d \ln(1+e^{2 f x+2 e})}{f^2}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{d x+c}{a+b \tanh(f x+e)}\,dx$$

Optimal(type 4, 107 leaves, 4 steps):

$$\frac{(d x+c)^2}{2 (a+b) d}-\frac{b (d x+c) \ln\left(1+\frac{a-b}{(a+b) e^{2 f x+2 e}}\right)}{(a^2-b^2) f}+\frac{b d \operatorname{polylog}\left(2,\frac{-a+b}{(a+b) e^{2 f x+2 e}}\right)}{2 (a^2-b^2) f^2}$$

Result(type 4, 356 leaves):

$$\frac{d x^2}{2 (a+b)} + \frac{c x}{a+b} - \frac{b c \ln\left(a e^{2 f x+2 e} + b e^{2 f x+2 e} + a - b\right)}{(a+b) f (a-b)} + \frac{2 b c \ln\left(e^{f x+e}\right)}{(a+b) f (a-b)} + \frac{b d \ln\left(1 - \frac{(a+b) e^{2 f x+2 e}}{-a+b}\right) x}{(a+b) f (-a+b)}$$
$$+ \frac{b d \ln\left(1 - \frac{(a+b) e^{2 f x+2 e}}{-a+b}\right) e}{(a+b) f^2 (-a+b)} - \frac{b d x^2}{(a+b) (-a+b)} - \frac{2 b d e x}{(a+b) f (-a+b)} - \frac{b d e^2}{(a+b) f^2 (-a+b)} + \frac{b d \operatorname{polylog}\left(2, \frac{(a+b) e^{2 f x+2 e}}{-a+b}\right)}{2 (a+b) f^2 (-a+b)}$$
$$+ \frac{b d e \ln\left(a e^{2 f x+2 e} + b e^{2 f x+2 e} + a - b\right)}{(a+b) f^2 (a-b)} - \frac{2 b d e \ln\left(e^{f x+e}\right)}{(a+b) f^2 (a-b)}$$

Test results for the 68 problems in "6.3.2 Hyperbolic tangent functions.txt"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \tanh(b x + a)^4 \, dx$$

Optimal(type 3, 26 leaves, 3 steps):

$$x - \frac{\tanh(b x + a)}{b} - \frac{\tanh(b x + a)^3}{3 b}$$

Result(type 3, 53 leaves):

$$-\frac{\tanh(b x + a)^3}{3 b} - \frac{\tanh(b x + a)}{b} - \frac{\ln(-1 + \tanh(b x + a))}{2 b} + \frac{\ln(1 + \tanh(b x + a))}{2 b}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \coth(b x + a) \, dx$$

Optimal(type 3, 11 leaves, 1 step):

$$\frac{\ln(\sinh(b x + a))}{b}$$

Result(type 3, 29 leaves):

$$-\frac{\ln(\coth(b x + a) - 1)}{2 b} - \frac{\ln(\coth(b x + a) + 1)}{2 b}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \coth(b x + a)^4 \, dx$$

Optimal(type 3, 26 leaves, 3 steps):

$$x - \frac{\coth(b x + a)}{b} - \frac{\coth(b x + a)^3}{3 b}$$

Result(type 3, 53 leaves):

$$-\frac{\coth(bx+a)^3}{3b}-\frac{\coth(bx+a)}{b}-\frac{\ln(\coth(bx+a)-1)}{2b}+\frac{\ln(\coth(bx+a)+1)}{2b}$$

Problem 10: Unable to integrate problem.

$$\int (b\tanh(dx+c))^n \, dx$$

Optimal(type 5, 46 leaves, 2 steps):

$$\frac{\mathrm{hypergeom}\left(\left[1, \frac{1}{2}+\frac{n}{2}\right], \left[\frac{3}{2}+\frac{n}{2}\right], \tanh(dx+c)^2\right) (b\tanh(dx+c))^{1+n}}{bd(1+n)}$$

Result(type 8, 12 leaves):

$$\int (b\tanh(dx+c))^n \, dx$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{a+a\tanh(dx+c)} \, dx$$

Optimal(type 3, 24 leaves, 2 steps):

$$\frac{x}{2a}-\frac{1}{2d(a+a\tanh(dx+c))}$$

Result(type 3, 53 leaves):

$$-\frac{\ln(\tanh(dx+c)-1)}{4ad}-\frac{1}{2ad(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{4ad}$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int (a+b\tanh(dx+c))^3 \, dx$$

Optimal(type 3, 67 leaves, 3 steps):

$$a(a^2+3b^2)x+\frac{b(3a^2+b^2)\ln(\cosh(dx+c))}{d}-\frac{2ab^2\tanh(dx+c)}{d}-\frac{b(a+b\tanh(dx+c))^2}{2d}$$

Result(type 3, 172 leaves):

$$-\frac{b^3\tanh(dx+c)^2}{2d}-\frac{3ab^2\tanh(dx+c)}{d}-\frac{\ln(\tanh(dx+c)-1)a^3}{2d}-\frac{3\ln(\tanh(dx+c)-1)a^2b}{2d}-\frac{3\ln(\tanh(dx+c)-1)ab^2}{2d}$$
$$-\frac{\ln(\tanh(dx+c)-1)b^3}{2d}+\frac{\ln(\tanh(dx+c)+1)a^3}{2d}-\frac{3\ln(\tanh(dx+c)+1)a^2b}{2d}+\frac{3\ln(\tanh(dx+c)+1)ab^2}{2d}-\frac{\ln(\tanh(dx+c)+1)b^3}{2d}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^3}{1+\tanh(x)}\, dx$$

Optimal(type 3, 19 leaves, 9 steps):

$$-\frac{\cosh(x)^3}{3}+\frac{\cosh(x)^5}{5}-\frac{\sinh(x)^5}{5}$$

Result(type 3, 71 leaves):

$$-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^5}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^4}{a+b\tanh(x)}\, dx$$

Optimal(type 3, 139 leaves, 5 steps):

$$-\frac{a(3a+b)\ln(1-\tanh(x))}{16(a+b)^3}+\frac{a(3a-b)\ln(1+\tanh(x))}{16(a-b)^3}-\frac{a^4 b\ln(a+b\tanh(x))}{\left(a^2-b^2\right)^3}-\frac{\cosh(x)^4(b-a\tanh(x))}{4\left(a^2-b^2\right)}+\frac{\cosh(x)^2\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\tanh(x)\right)}{8\left(a^2-b^2\right)^2}$$

Result(type 3, 319 leaves):

$$\frac{8}{(32a+32b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{32}{(64a+64b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{a}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{3a}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3a^2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8(a+b)^3}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b}{8(a+b)^3}-\frac{b a^4\ln\left(\tanh\left(\frac{x}{2}\right)^2 a+2\tanh\left(\frac{x}{2}\right)b+a\right)}{(a+b)^3(a-b)^3}-\frac{8}{(32a-32b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{32}{(64a-64b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{a}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{3a}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}$$

$$+\frac{3 a^2 \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8 (a-b)^3}-\frac{a \ln\left(\tanh\left(\frac{x}{2}\right)+1\right) b}{8 (a-b)^3}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(x)^4}{1+\tanh(x)}\, dx$$

Optimal(type 3, 48 leaves, 4 steps):

$$\frac{5 x}{16}+\frac{1}{32 (1-\tanh(x))^2}+\frac{1}{8 (1-\tanh(x))}-\frac{1}{24 (1+\tanh(x))^3}-\frac{3}{32 (1+\tanh(x))^2}-\frac{3}{16 (1+\tanh(x))}$$

Result(type 3, 115 leaves):

$$\frac{1}{8 \left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{1}{4 \left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{1}{2 \left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{3}{8 \left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{5 \ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16}-\frac{1}{3 \left(\tanh\left(\frac{x}{2}\right)+1\right)^6}$$
$$+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^5}-\frac{15}{8 \left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{25}{12 \left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{15}{8 \left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{5 \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{16}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(x)^3}{1+\tanh(x)}\, dx$$

Optimal(type 3, 23 leaves, 3 steps):

$$\frac{4 \sinh(x)}{5}+\frac{4 \sinh(x)^3}{15}-\frac{\cosh(x)^3}{5 (1+\tanh(x))}$$

Result(type 3, 79 leaves):

$$-\frac{1}{6 \left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{4 \left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{5}{8 \left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2}{5 \left(\tanh\left(\frac{x}{2}\right)+1\right)^5}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{5}{3 \left(\tanh\left(\frac{x}{2}\right)+1\right)^3}$$
$$+\frac{3}{2 \left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{11}{8 \left(\tanh\left(\frac{x}{2}\right)+1\right)}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^6}{1+\tanh(x)}\, dx$$

Optimal(type 3, 21 leaves, 3 steps):

$$-\frac{2\,(1-\tanh(x))^3}{3}+\frac{(1-\tanh(x))^4}{4}$$

Result(type 3, 55 leaves):

$$-\frac{2\left(-\tanh\left(\frac{x}{2}\right)^7+\tanh\left(\frac{x}{2}\right)^6-\frac{5\tanh\left(\frac{x}{2}\right)^5}{3}-\frac{5\tanh\left(\frac{x}{2}\right)^3}{3}+\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^4}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{sech}(x)^7}{1+\tanh(x)}\,dx$$

Optimal(type 3, 26 leaves, 4 steps):

$$\frac{3\arctan(\sinh(x))}{8}+\frac{\mathrm{sech}(x)^5}{5}+\frac{3\,\mathrm{sech}(x)\tanh(x)}{8}+\frac{\mathrm{sech}(x)^3\tanh(x)}{4}$$

Result(type 3, 66 leaves):

$$\frac{2\left(-\frac{5\tanh\left(\frac{x}{2}\right)^9}{8}+\tanh\left(\frac{x}{2}\right)^8-\frac{\tanh\left(\frac{x}{2}\right)^7}{4}+2\tanh\left(\frac{x}{2}\right)^4+\frac{\tanh\left(\frac{x}{2}\right)^3}{4}+\frac{5\tanh\left(\frac{x}{2}\right)}{8}+\frac{1}{5}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^5}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{sech}(x)^8}{a+b\tanh(x)}\,dx$$

Optimal(type 3, 130 leaves, 3 steps):

$$-\frac{(a^2-b^2)^3\ln(a+b\tanh(x))}{b^7}+\frac{a\,(a^4-3a^2b^2+3b^4)\tanh(x)}{b^6}-\frac{(a^4-3a^2b^2+3b^4)\tanh(x)^2}{2b^5}+\frac{a\,(a^2-3b^2)\tanh(x)^3}{3b^4}-\frac{(a^2-3b^2)\tanh(x)^4}{4b^3}+\frac{a\tanh(x)^5}{5b^2}-\frac{\tanh(x)^6}{6b}$$

Result(type 3, 924 leaves):

$$\frac{10\tanh\left(\frac{x}{2}\right)^9a^5}{b^6\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^6}-\frac{82\tanh\left(\frac{x}{2}\right)^9a^3}{3b^4\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^6}+\frac{22\tanh\left(\frac{x}{2}\right)^9a}{b^2\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^6}+\frac{20\tanh\left(\frac{x}{2}\right)^7a^5}{b^6\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^6}-\frac{52\tanh\left(\frac{x}{2}\right)^7a^3}{b^4\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^6}$$

$$+\frac{212\tanh\left(\frac{x}{2}\right)^{7}a}{5b^{2}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{12\tanh\left(\frac{x}{2}\right)^{6}a^{4}}{b^{5}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{28\tanh\left(\frac{x}{2}\right)^{6}a^{2}}{b^{3}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{20\tanh\left(\frac{x}{2}\right)^{5}a^{5}}{b^{6}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{52\tanh\left(\frac{x}{2}\right)^{5}a^{3}}{b^{4}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}$$

$$+\frac{212\tanh\left(\frac{x}{2}\right)^{5}a}{5b^{2}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{10\tanh\left(\frac{x}{2}\right)^{3}a^{5}}{b^{6}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{82\tanh\left(\frac{x}{2}\right)^{3}a^{3}}{3b^{4}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{22\tanh\left(\frac{x}{2}\right)^{3}a}{b^{2}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{2\tanh\left(\frac{x}{2}\right)^{11}a^{5}}{b^{6}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}$$

$$-\frac{6\tanh\left(\frac{x}{2}\right)^{11}a^{3}}{b^{4}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{6\tanh\left(\frac{x}{2}\right)^{11}a}{b^{2}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{2\tanh\left(\frac{x}{2}\right)^{10}a^{4}}{b^{5}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{6\tanh\left(\frac{x}{2}\right)^{10}a^{2}}{b^{3}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{8\tanh\left(\frac{x}{2}\right)^{8}a^{4}}{b^{5}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}$$

$$+\frac{20\tanh\left(\frac{x}{2}\right)^{8}a^{2}}{b^{3}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{8\tanh\left(\frac{x}{2}\right)^{4}a^{4}}{b^{5}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{20\tanh\left(\frac{x}{2}\right)^{4}a^{2}}{b^{3}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{2\tanh\left(\frac{x}{2}\right)^{2}a^{4}}{b^{5}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{6\tanh\left(\frac{x}{2}\right)^{2}a^{2}}{b^{3}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}$$

$$+\frac{2\tanh\left(\frac{x}{2}\right)a^{5}}{b^{6}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{6\tanh\left(\frac{x}{2}\right)a^{3}}{b^{4}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}+\frac{6\tanh\left(\frac{x}{2}\right)a}{b^{2}\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)a^{4}}{b^{5}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)a^{2}}{b^{3}}$$

$$-\frac{68\tanh\left(\frac{x}{2}\right)^{6}}{3b\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{6\tanh\left(\frac{x}{2}\right)^{10}}{b\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{12\tanh\left(\frac{x}{2}\right)^{8}}{b\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{12\tanh\left(\frac{x}{2}\right)^{4}}{b\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}-\frac{6\tanh\left(\frac{x}{2}\right)^{2}}{b\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{6}}$$

$$+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)a^{6}}{b^{7}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a+2\tanh\left(\frac{x}{2}\right)b+a\right)a^{6}}{b^{7}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a+2\tanh\left(\frac{x}{2}\right)b+a\right)a^{4}}{b^{5}}$$

$$-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a+2\tanh\left(\frac{x}{2}\right)b+a\right)a^{2}}{b^{3}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a+2\tanh\left(\frac{x}{2}\right)b+a\right)}{b}$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int\frac{\coth(x)^{2}}{1+\tanh(x)}\,dx$$

Optimal(type 3, 23 leaves, 4 steps):

$$\frac{3x}{2}-\frac{3\coth(x)}{2}-\ln(\sinh(x))+\frac{\coth(x)}{2\left(1+\tanh(x)\right)}$$

Result(type 3, 58 leaves):

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(x)^{4}}{1+\tanh(x)}\,dx$$

Optimal(type 3, 35 leaves, 6 steps):

$$\frac{5x}{2}-\frac{5\coth(x)}{2}+\coth(x)^{2}-\frac{5\coth(x)^{3}}{6}-2\ln(\sinh(x))+\frac{\coth(x)^{3}}{2\,(1+\tanh(x))}$$

Result(type 3, 90 leaves):

$$-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{24}+\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8}-\frac{9\tanh\left(\frac{x}{2}\right)}{8}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}+\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}-\frac{9}{8\tanh\left(\frac{x}{2}\right)}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$
$$-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{9\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}$$

Problem 42: Result is not expressed in closed-form.

$$\int x^{2}\tanh(a+2\ln(x))\,dx$$

Optimal(type 3, 111 leaves, 11 steps):

$$\frac{x^{3}}{3}-\frac{\arctan\left(e^{\frac{a}{2}}x\sqrt{2}-1\right)\sqrt{2}}{2\,e^{\frac{3a}{2}}}-\frac{\arctan\left(1+e^{\frac{a}{2}}x\sqrt{2}\right)\sqrt{2}}{2\,e^{\frac{3a}{2}}}-\frac{\ln\left(1+e^{a}x^{2}-e^{\frac{a}{2}}x\sqrt{2}\right)\sqrt{2}}{4\,e^{\frac{3a}{2}}}+\frac{\ln\left(1+e^{a}x^{2}+e^{\frac{a}{2}}x\sqrt{2}\right)\sqrt{2}}{4\,e^{\frac{3a}{2}}}$$

Result(type 7, 36 leaves):

$$\frac{x^{3}}{3}-\frac{e^{-2a}\left(\sum_{\_R=RootOf(e^{2a}\_Z^{4}+1)}\frac{\ln(x-\_R)}{\_R}\right)}{2}$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int x\tanh(a+2\ln(x))\,dx$$

Optimal(type 3, 19 leaves, 4 steps):

$$\frac{x^2}{2}-\frac{\arctan\left(e^a x^2\right)}{e^a}$$

Result(type 3, 40 leaves):

$$\frac{x^2}{2}+\frac{I\, e^{-a} \ln\left(e^a x^2-I\right)}{2}-\frac{I\, e^{-a} \ln\left(e^a x^2+I\right)}{2}$$

Problem 44: Result is not expressed in closed-form.

$$\int \frac{\tanh(a+2\ln(x))}{x^3}\, dx$$

Optimal(type 3, 16 leaves, 4 steps):

$$\frac{1}{2x^2}+e^a \arctan\left(e^a x^2\right)$$

Result(type 7, 43 leaves):

$$\frac{1}{2x^2}+\frac{\left(\sum_{\_R=RootOf(e^{2a}+\_Z^2)} \_R \ln\left(\left(4e^{2a}+5\_R^2\right)x^2-\_R\right)\right)}{2}$$

Problem 46: Result is not expressed in closed-form.

$$\int \tanh(a+2\ln(x))^2\, dx$$

Optimal(type 3, 121 leaves, 13 steps):

$$x+\frac{x}{1+e^{2a}x^4}-\frac{\arctan\left(e^{\frac{a}{2}} x\sqrt{2}-1\right)\sqrt{2}}{4e^{\frac{a}{2}}}-\frac{\arctan\left(1+e^{\frac{a}{2}} x\sqrt{2}\right)\sqrt{2}}{4e^{\frac{a}{2}}}+\frac{\ln\left(1+e^a x^2-e^{\frac{a}{2}} x\sqrt{2}\right)\sqrt{2}}{8e^{\frac{a}{2}}}-\frac{\ln\left(1+e^a x^2+e^{\frac{a}{2}} x\sqrt{2}\right)\sqrt{2}}{8e^{\frac{a}{2}}}$$

Result(type 7, 46 leaves):

$$x+\frac{x}{1+e^{2a}x^4}-\frac{e^{-2a}\left(\sum_{\_R=RootOf(e^{2a}\_Z^4+1)} \frac{\ln(x-\_R)}{\_R^3}\right)}{4}$$

Problem 47: Unable to integrate problem.

$$\int \tanh\left(a+\frac{\ln(x)}{2}\right)^p dx$$

Optimal(type 5, 48 leaves, 2 steps):

$$\frac{(-1+e^{2a}x)^{1+p}\,\text{hypergeom}\left([p, 1+p], [2+p], \frac{1}{2}-\frac{e^{2a}x}{2}\right)}{2^p e^{2a}(1+p)}$$

Result(type 8, 11 leaves):

$$\int \tanh\left(a+\frac{\ln(x)}{2}\right)^p dx$$

Problem 48: Unable to integrate problem.

$$\int \tanh\left(a+\frac{\ln(x)}{6}\right)^p dx$$

Optimal(type 5, 139 leaves, 5 steps):

$$-\frac{p\,(-1+e^{2a}x^{1/3})^{1+p}(1+e^{2a}x^{1/3})^{1-p}}{e^{6a}}+\frac{(-1+e^{2a}x^{1/3})^{1+p}(1+e^{2a}x^{1/3})^{1-p}x^{1/3}}{e^{4a}}$$
$$+\frac{(2p^2+1)(-1+e^{2a}x^{1/3})^{1+p}\,\text{hypergeom}\left([p, 1+p], [2+p], \frac{1}{2}-\frac{e^{2a}x^{1/3}}{2}\right)}{2^p e^{6a}(1+p)}$$

Result(type 8, 11 leaves):

$$\int \tanh\left(a+\frac{\ln(x)}{6}\right)^p dx$$

Problem 50: Unable to integrate problem.

$$\int (xe)^m \tanh(d(a+b\ln(cx^n)))^2 dx$$

Optimal(type 5, 164 leaves, 5 steps):

$$\frac{(dbn+m+1)(xe)^{1+m}}{bde(1+m)n}+\frac{(xe)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bden\left(1+e^{2ad}(cx^n)^{2bd}\right)}-\frac{2(xe)^{1+m}\,\text{hypergeom}\left(\left[1, \frac{1+m}{2bdn}\right], \left[1+\frac{1+m}{2bdn}\right], -e^{2ad}(cx^n)^{2bd}\right)}{bden}$$

Result(type 8, 326 leaves):

$$\frac{x\,e^{m\left(\ln(e)+\ln(x)-\frac{I\pi\,\text{csgn}(Iex)(-\text{csgn}(Iex)+\text{csgn}(Ie))(-\text{csgn}(Iex)+\text{csgn}(Ix))}{2}\right)}}{1+m}$$
$$+\frac{2x\,e^{m\left(\ln(e)+\ln(x)-\frac{I\pi\,\text{csgn}(Iex)(-\text{csgn}(Iex)+\text{csgn}(Ie))(-\text{csgn}(Iex)+\text{csgn}(Ix))}{2}\right)}}{dbn\left(\left(e^{d\left(a+b\left(\ln(c)+\ln(e^{n\ln(x)})-\frac{I\pi\,\text{csgn}(Ice^{n\ln(x)})(-\text{csgn}(Ice^{n\ln(x)})+\text{csgn}(Ic))(-\text{csgn}(Ice^{n\ln(x)})+\text{csgn}(Ie^{n\ln(x)}))}{2}\right)\right)}\right)^2+1\right)}+\int$$

$$- \frac{2\, e^{m \left(\ln(e) + \ln(x) - \frac{I \pi \operatorname{csgn}(I e x) (-\operatorname{csgn}(I e x) + \operatorname{csgn}(I e)) (-\operatorname{csgn}(I e x) + \operatorname{csgn}(I x))}{2}\right)} (1+m)}{d\, b\, n \left(\left(e^{d \left(a + b \left(\ln(c) + \ln(e^{n \ln(x)}) - \frac{I \pi \operatorname{csgn}(I c e^{n \ln(x)}) (-\operatorname{csgn}(I c e^{n \ln(x)}) + \operatorname{csgn}(I c)) (-\operatorname{csgn}(I c e^{n \ln(x)}) + \operatorname{csgn}(I e^{n \ln(x)}))}{2}\right)\right)} \right)^2 + 1 \right)} \, dx$$

Problem 53: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4} \tanh(x) \, dx$$

Optimal(type 3, 108 leaves, 8 steps):

$$- \frac{(b+2c) \operatorname{arctanh}\left(\frac{b + 2 c \tanh(x)^2}{2 \sqrt{c} \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}}\right)}{4 \sqrt{c}} + \frac{\operatorname{arctanh}\left(\frac{2a + b + (b+2c) \tanh(x)^2}{2 \sqrt{a+b+c} \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}}\right) \sqrt{a+b+c}}{2} - \frac{\sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}}{2}$$

Result(type 4, 558 leaves):

$$- \frac{\sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}}{2} - \frac{1}{8 \sqrt{\frac{-b + \sqrt{-4ac + b^2}}{a}} \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}} \Bigg((b + c) \sqrt{2} \sqrt{4 - \frac{2 \left(-b + \sqrt{-4ac + b^2}\right) \tanh(x)^2}{a}} \sqrt{4 + \frac{2 \left(b + \sqrt{-4ac + b^2}\right) \tanh(x)^2}{a}} \operatorname{EllipticF}\left(\frac{\tanh(x) \sqrt{2} \sqrt{\frac{-b + \sqrt{-4ac + b^2}}{a}}}{2}, \frac{\sqrt{-4 + \frac{2b \left(b + \sqrt{-4ac + b^2}\right)}{ac}}}{2}\right) \Bigg) - \frac{\ln\left(\frac{b + 2c \tanh(x)^2}{\sqrt{c}} + 2 \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}\right) b}{4 \sqrt{c}}$$

$$- \frac{\ln\left(\frac{b + 2c \tanh(x)^2}{\sqrt{c}} + 2 \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}\right) \sqrt{c}}{2} + \frac{a \operatorname{arctanh}\left(\frac{b \tanh(x)^2 + 2c \tanh(x)^2 + 2a + b}{2 \sqrt{a+b+c} \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}}\right)}{2 \sqrt{a+b+c}}$$

$$+ \frac{b \operatorname{arctanh}\left(\frac{b \tanh(x)^2 + 2c \tanh(x)^2 + 2a + b}{2 \sqrt{a+b+c} \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}}\right)}{2 \sqrt{a+b+c}} + \frac{c \operatorname{arctanh}\left(\frac{b \tanh(x)^2 + 2c \tanh(x)^2 + 2a + b}{2 \sqrt{a+b+c} \sqrt{a + b \tanh(x)^2 + c \tanh(x)^4}}\right)}{2 \sqrt{a+b+c}}$$

$$-\frac{1}{8\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}\Bigg((-b$$

$$-c)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)\tanh(x)^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)\tanh(x)^2}{a}}\,\mathrm{EllipticF}\left(\frac{\tanh(x)\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.$$

$$\left.\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)\Bigg)$$

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Problem 57: Result more than twice size of optimal antiderivative.
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$$\int e^x \coth(3x)\,dx$$

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Optimal(type 3, 66 leaves, 12 steps):
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$$e^x-\frac{2\,\mathrm{arctanh}(e^x)}{3}+\frac{\ln(1-e^x+e^{2x})}{6}-\frac{\ln(1+e^x+e^{2x})}{6}+\frac{\arctan\left(\frac{(1-2e^x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\arctan\left(\frac{(1+2e^x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$

```
Result(type 3, 137 leaves):
```

$$e^x-\frac{\ln\left(e^x+\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)}{6}+\frac{I\ln\left(e^x+\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{\ln\left(e^x+\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)}{6}-\frac{I\ln\left(e^x+\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{\ln(e^x+1)}{3}+\frac{\ln(e^x-1)}{3}$$

$$+\frac{\ln\left(e^x-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)}{6}+\frac{I\ln\left(e^x-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(e^x-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)}{6}-\frac{I\ln\left(e^x-\frac{1}{2}+\frac{I\sqrt{3}}{2}\right)\sqrt{3}}{6}$$

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Problem 59: Result is not expressed in closed-form.
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$$\int e^x \coth(4x)^2\,dx$$

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Optimal(type 3, 92 leaves, 17 steps):
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$$e^x+\frac{e^x}{2(1-e^{8x})}-\frac{\arctan(e^x)}{8}-\frac{\mathrm{arctanh}(e^x)}{8}-\frac{\arctan\left(e^x\sqrt{2}-1\right)\sqrt{2}}{16}-\frac{\arctan\left(1+e^x\sqrt{2}\right)\sqrt{2}}{16}+\frac{\ln\left(1+e^{2x}-e^x\sqrt{2}\right)\sqrt{2}}{32}$$

$$-\frac{\ln\left(1+e^{2x}+e^x\sqrt{2}\right)\sqrt{2}}{32}$$

```
Result(type 7, 67 leaves):
```

$$e^{x} - \frac{e^{x}}{2\left(e^{8x}-1\right)} + \left(\sum_{\_R = RootOf\left(65536\,\_Z^{4}+1\right)} \_R \ln\left(e^{x} - 16\,\_R\right)\right) - \frac{\ln\left(e^{x}+1\right)}{16} + \frac{\ln\left(e^{x}-1\right)}{16} + \frac{I\ln\left(e^{x}-I\right)}{16} - \frac{I\ln\left(e^{x}+I\right)}{16}$$

Problem 60: Unable to integrate problem.

$$\int e^{c(bx+a)}\tanh(xe+d)^{3}\,dx$$

Optimal(type 5, 157 leaves, 6 steps):

$$\frac{e^{c(bx+a)}}{bc} - \frac{6\,e^{c(bx+a)}\,\mathrm{hypergeom}\left(\left[1, \frac{bc}{2e}\right], \left[1+\frac{bc}{2e}\right], -e^{2xe+2d}\right)}{bc} + \frac{12\,e^{c(bx+a)}\,\mathrm{hypergeom}\left(\left[2, \frac{bc}{2e}\right], \left[1+\frac{bc}{2e}\right], -e^{2xe+2d}\right)}{bc} - \frac{8\,e^{c(bx+a)}\,\mathrm{hypergeom}\left(\left[3, \frac{bc}{2e}\right], \left[1+\frac{bc}{2e}\right], -e^{2xe+2d}\right)}{bc}$$

Result(type 8, 19 leaves):

$$\int e^{c(bx+a)}\tanh(xe+d)^{3}\,dx$$

Problem 61: Unable to integrate problem.

$$\int e^{c(bx+a)}\tanh(xe+d)\,dx$$

Optimal(type 5, 63 leaves, 4 steps):

$$\frac{e^{c(bx+a)}}{bc} - \frac{2\,e^{c(bx+a)}\,\mathrm{hypergeom}\left(\left[1, \frac{bc}{2e}\right], \left[1+\frac{bc}{2e}\right], -e^{2xe+2d}\right)}{bc}$$

Result(type 8, 17 leaves):

$$\int e^{c(bx+a)}\tanh(xe+d)\,dx$$

Problem 62: Unable to integrate problem.

$$\int e^{c(bx+a)}\coth(xe+d)\,dx$$

Optimal(type 5, 61 leaves, 4 steps):

$$\frac{e^{c(bx+a)}}{bc} - \frac{2\,e^{c(bx+a)}\,\mathrm{hypergeom}\left(\left[1, \frac{bc}{2e}\right], \left[1+\frac{bc}{2e}\right], e^{2xe+2d}\right)}{bc}$$

Result(type 8, 17 leaves):

$$\int e^{c(bx+a)}\coth(xe+d)\,dx$$

Problem 63: Unable to integrate problem.

$$\int e^{c (b x+a)} \coth(x e+d)^3 \, dx$$

Optimal (type 5, 151 leaves, 6 steps):

$$\frac{e^{c (b x+a)}}{b c}-\frac{6 e^{c (b x+a)} \operatorname{hypergeom}\left(\left[1, \frac{b c}{2 e}\right],\left[1+\frac{b c}{2 e}\right], e^{2 x e+2 d}\right)}{b c}+\frac{12 e^{c (b x+a)} \operatorname{hypergeom}\left(\left[2, \frac{b c}{2 e}\right],\left[1+\frac{b c}{2 e}\right], e^{2 x e+2 d}\right)}{b c}-\frac{8 e^{c (b x+a)} \operatorname{hypergeom}\left(\left[3, \frac{b c}{2 e}\right],\left[1+\frac{b c}{2 e}\right], e^{2 x e+2 d}\right)}{b c}$$

Result (type 8, 19 leaves):

$$\int e^{c (b x+a)} \coth(x e+d)^3 \, dx$$

Test results for the 71 problems in "6.3.7 (d hyper)^m (a+b (c tanh)^n)^p.txt"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \sinh(d x+c)^3 \left(a+b \tanh(d x+c)^2\right)^2 dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$-\frac{(a+b) (a+3 b) \cosh(d x+c)}{d}+\frac{(a+b)^2 \cosh(d x+c)^3}{3 d}-\frac{b (2 a+3 b) \operatorname{sech}(d x+c)}{d}+\frac{b^2 \operatorname{sech}(d x+c)^3}{3 d}$$

Result (type 3, 161 leaves):

$$\frac{1}{d}\left(a^2\left(-\frac{2}{3}+\frac{\sinh(d x+c)^2}{3}\right) \cosh(d x+c)+2 b a\left(\frac{\sinh(d x+c)^4}{3 \cosh(d x+c)}+\frac{4 \sinh(d x+c)^2}{3 \cosh(d x+c)}-\frac{8 \cosh(d x+c)}{3}\right)+b^2\left(\frac{\sinh(d x+c)^6}{3 \cosh(d x+c)^3}-\frac{2 \sinh(d x+c)^4}{\cosh(d x+c)^3}-\frac{8 \sinh(d x+c)^2}{3 \cosh(d x+c)^3}+\frac{16 \sinh(d x+c)^2}{3 \cosh(d x+c)}-\frac{16 \cosh(d x+c)}{3}\right)\right)$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \sinh(d x+c)^3 \left(a+b \tanh(d x+c)^2\right)^3 dx$$

Optimal (type 3, 99 leaves, 3 steps):

$$-\frac{(a+b)^2 (a+4 b) \cosh(d x+c)}{d}+\frac{(a+b)^3 \cosh(d x+c)^3}{3 d}-\frac{3 b (a+b) (a+2 b) \operatorname{sech}(d x+c)}{d}+\frac{b^2 (3 a+4 b) \operatorname{sech}(d x+c)^3}{3 d}-\frac{b^3 \operatorname{sech}(d x+c)^5}{5 d}$$

Result (type 3, 286 leaves):

$$\frac{1}{d}\left(a^3\left(-\frac{2}{3}+\frac{\sinh(d x+c)^2}{3}\right) \cosh(d x+c)+3 a^2 b\left(\frac{\sinh(d x+c)^4}{3 \cosh(d x+c)}+\frac{4 \sinh(d x+c)^2}{3 \cosh(d x+c)}-\frac{8 \cosh(d x+c)}{3}\right)+3 a b^2\left(\frac{\sinh(d x+c)^6}{3 \cosh(d x+c)^3}\right.\right.$$

$$- \frac{2 \sinh(dx+c)^4}{\cosh(dx+c)^3} - \frac{8 \sinh(dx+c)^2}{3 \cosh(dx+c)^3} + \frac{16 \sinh(dx+c)^2}{3 \cosh(dx+c)} - \frac{16 \cosh(dx+c)}{3} \Bigg) + b^3 \Bigg(\frac{\sinh(dx+c)^8}{3 \cosh(dx+c)^5} - \frac{8 \sinh(dx+c)^6}{3 \cosh(dx+c)^5} - \frac{16 \sinh(dx+c)^4}{\cosh(dx+c)^5}$$

$$- \frac{64 \sinh(dx+c)^2}{5 \cosh(dx+c)^5} + \frac{128 \sinh(dx+c)^2}{15 \cosh(dx+c)^3} + \frac{128 \sinh(dx+c)^2}{15 \cosh(dx+c)} - \frac{128 \cosh(dx+c)}{15} \Bigg) \Bigg)$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(dx+c)^2}{\left(a+b\tanh(dx+c)^2\right)^2}\, dx$$

Optimal(type 3, 118 leaves, 6 steps):

$$-\frac{(a-3b)\,x}{2\,(a+b)^3} - \frac{(3a-b)\arctan\left(\frac{\sqrt{b}\,\tanh(dx+c)}{\sqrt{a}}\right)\sqrt{b}}{2\,(a+b)^3\, d\sqrt{a}} + \frac{\cosh(dx+c)\sinh(dx+c)}{2\,(a+b)\,d\,\left(a+b\tanh(dx+c)^2\right)} - \frac{b\tanh(dx+c)}{(a+b)^2\, d\,\left(a+b\tanh(dx+c)^2\right)}$$

Result(type 3, 1127 leaves):

$$\frac{1}{2\,d\,(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{1}{2\,d\,(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a}{2\,d\,(a+b)^3} - \frac{3\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b}{2\,d\,(a+b)^3}$$

$$- \frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{d\,(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 + 2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a + 4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 + a\right)}$$

$$- \frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d\,(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 + 2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a + 4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 + a\right)}$$

$$- \frac{b\,a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 + 2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a + 4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 + a\right)}$$

$$- \frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 + 2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a + 4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 + a\right)} + \frac{3\,b\,a^2\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b\,(a+b)}+a+2\,b\right)a}}\right)}{2\,d\,(a+b)^3\sqrt{b\,(a+b)}\,\sqrt{\left(2\sqrt{b\,(a+b)}+a+2\,b\right)a}}$$

$$+\frac{3 b a \arctan\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}+a+2 b\right) a}}\right)}{2 d (a+b)^{3}\sqrt{\left(2\sqrt{b (a+b)}+a+2 b\right) a}}+\frac{b^{2} a \arctan\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}+a+2 b\right) a}}\right)}{d (a+b)^{3}\sqrt{b (a+b)}\sqrt{\left(2\sqrt{b (a+b)}+a+2 b\right) a}}$$

$$+\frac{3 b a^{2} \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}\right)}{2 d (a+b)^{3}\sqrt{b (a+b)}\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}-\frac{3 b a \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}\right)}{2 d (a+b)^{3}\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}$$

$$+\frac{b^{2} a \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}\right)}{d (a+b)^{3}\sqrt{b (a+b)}\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}-\frac{b^{2} \arctan\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}+a+2 b\right) a}}\right)}{2 d (a+b)^{3}\sqrt{\left(2\sqrt{b (a+b)}+a+2 b\right) a}}$$

$$-\frac{b^{3} \arctan\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}+a+2 b\right) a}}\right)}{2 d (a+b)^{3}\sqrt{b (a+b)}\sqrt{\left(2\sqrt{b (a+b)}+a+2 b\right) a}}+\frac{b^{2} \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}\right)}{2 d (a+b)^{3}\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}$$

$$-\frac{b^{3} \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}\right)}{2 d (a+b)^{3}\sqrt{b (a+b)}\sqrt{\left(2\sqrt{b (a+b)}-a-2 b\right) a}}-\frac{1}{2 d (a+b)^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{2 d (a+b)^{2}\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}$$

$$-\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right) a}{2 d (a+b)^{3}}+\frac{3 \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right) b}{2 d (a+b)^{3}}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int\frac{\operatorname{csch}(d x+c)^{3}}{\left(a+b\tanh(d x+c)^{2}\right)^{2}}\, dx$$

Optimal(type 3, 127 leaves, 6 steps):

$$\frac{(a+4 b) \operatorname{arctanh}(\cosh(d x+c))}{2 a^{3} d}-\frac{\coth(d x+c) \operatorname{csch}(d x+c)}{2 a d \left(a+b-b \operatorname{sech}(d x+c)^{2}\right)}-\frac{b \operatorname{sech}(d x+c)}{a^{2} d \left(a+b-b \operatorname{sech}(d x+c)^{2}\right)}-\frac{(3 a+4 b) \operatorname{arctanh}\left(\frac{\operatorname{sech}(d x+c)\sqrt{b}}{\sqrt{a+b}}\right)\sqrt{b}}{2 a^{3} d\sqrt{a+b}}$$

Result(type 3, 366 leaves):

$$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8\,d\,a^2}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\,a^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}$$

$$-\frac{2\,b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\,a^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}$$

$$-\frac{b}{d\,a^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}-\frac{3\,b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2\,a+4\,b}{4\sqrt{b\,a+b^2}}\right)}{2\,d\,a^2\sqrt{b\,a+b^2}}$$

$$-\frac{2\,b^2\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2\,a+4\,b}{4\sqrt{b\,a+b^2}}\right)}{d\,a^3\sqrt{b\,a+b^2}}-\frac{1}{8\,d\,a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\,d\,a^2}-\frac{2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d\,a^3}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int\frac{\sinh(dx+c)^4}{\left(a+b\tanh(dx+c)^2\right)^3}\,dx$$

Optimal(type 3, 220 leaves, 8 steps):

$$\frac{3\left(a^2-10\,b\,a+5\,b^2\right)x}{8\,(a+b)^5}+\frac{3\left(5\,a^2-10\,b\,a+b^2\right)\arctan\left(\frac{\sqrt{b}\,\tanh(dx+c)}{\sqrt{a}}\right)\sqrt{b}}{8\,(a+b)^5\,d\sqrt{a}}-\frac{(5\,a-3\,b)\cosh(dx+c)\sinh(dx+c)}{8\,(a+b)^2\,d\left(a+b\tanh(dx+c)^2\right)^2}$$

$$+\frac{\cosh(dx+c)^3\sinh(dx+c)}{4\,(a+b)\,d\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(7\,a-5\,b)\,b\tanh(dx+c)}{8\,(a+b)^3\,d\left(a+b\tanh(dx+c)^2\right)^2}+\frac{3\,(a-b)\,b\tanh(dx+c)}{2\,(a+b)^4\,d\left(a+b\tanh(dx+c)^2\right)}$$

Result(type ?, 2365 leaves): Display of huge result suppressed!

Problem 15: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{csch}(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^3}\,dx$$

Optimal(type 3, 142 leaves, 6 steps):

$$-\frac{\mathrm{arctanh}(\cosh(dx+c))}{a^3\,d}+\frac{b\,\mathrm{sech}(dx+c)}{4\,a\,(a+b)\,d\left(a+b-b\,\mathrm{sech}(dx+c)^2\right)^2}+\frac{b\,(7\,a+4\,b)\,\mathrm{sech}(dx+c)}{8\,a^2\,(a+b)^2\,d\left(a+b-b\,\mathrm{sech}(dx+c)^2\right)}$$

$$+\frac{\left(15 a^{2}+20 b a+8 b^{2}\right) \operatorname{arctanh}\left(\frac{\operatorname{sech}(d x+c) \sqrt{b}}{\sqrt{a+b}}\right) \sqrt{b}}{8 a^{3} (a+b)^{5/2} d}$$

Result(type 3, 1131 leaves):

$$\frac{9 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{4 d \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} \left(a^{2}+2 b a+b^{2}\right)}$$

$$+\frac{7 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{d a \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} \left(a^{2}+2 b a+b^{2}\right)}$$

$$+\frac{4 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{d a^{2} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} \left(a^{2}+2 b a+b^{2}\right)}$$

$$+\frac{27 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{4 d \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} \left(a^{2}+2 b a+b^{2}\right)}$$

$$+\frac{45 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{2 d a \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} \left(a^{2}+2 b a+b^{2}\right)}$$

$$+\frac{30 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d a^{2} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} \left(a^{2}+2 b a+b^{2}\right)}$$

$$+\frac{12 b^{4} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d a^{3} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} \left(a^{2}+2 b a+b^{2}\right)}$$

$$+\frac{27 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{4 d \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} \left(a^{2}+2 b a+b^{2}\right)}$$

$$+\frac{17 b^2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{d a \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+a\right)^2 \left(a^2+2 b a+b^2\right)}$$

$$+\frac{8 b^3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2}{d a^2 \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+a\right)^2 \left(a^2+2 b a+b^2\right)}$$

$$+\frac{9 b}{4 d \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+a\right)^2 \left(a^2+2 b a+b^2\right)}$$

$$+\frac{3 b^2}{2 d a \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+a\right)^2 \left(a^2+2 b a+b^2\right)}$$

$$+\frac{15 b \operatorname{arctanh}\left(\frac{2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+2 a+4 b}{4 \sqrt{b a+b^2}}\right)}{8 d a \left(a^2+2 b a+b^2\right) \sqrt{b a+b^2}}+\frac{5 b^2 \operatorname{arctanh}\left(\frac{2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+2 a+4 b}{4 \sqrt{b a+b^2}}\right)}{2 d a^2 \left(a^2+2 b a+b^2\right) \sqrt{b a+b^2}}$$

$$+\frac{b^3 \operatorname{arctanh}\left(\frac{2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+2 a+4 b}{4 \sqrt{b a+b^2}}\right)}{d a^3 \left(a^2+2 b a+b^2\right) \sqrt{b a+b^2}}+\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d a^3}$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{csch}(d x+c)^3}{\left(a+b \tanh(d x+c)^2\right)^3}\, dx$$

Optimal(type 3, 178 leaves, 7 steps):

$$\frac{(a+6 b) \operatorname{arctanh}(\cosh(d x+c))}{2 a^4 d}-\frac{\coth(d x+c) \operatorname{csch}(d x+c)}{2 a d \left(a+b-b \operatorname{sech}(d x+c)^2\right)^2}-\frac{3 b \operatorname{sech}(d x+c)}{4 a^2 d \left(a+b-b \operatorname{sech}(d x+c)^2\right)^2}-\frac{b (11 a+12 b) \operatorname{sech}(d x+c)}{8 a^3 (a+b) d \left(a+b-b \operatorname{sech}(d x+c)^2\right)}$$

$$-\frac{\left(15 a^2+40 b a+24 b^2\right) \operatorname{arctanh}\left(\frac{\operatorname{sech}(d x+c) \sqrt{b}}{\sqrt{a+b}}\right) \sqrt{b}}{8 a^4 (a+b)^{3/2} d}$$

Result(type 3, 1082 leaves):

$$\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{8 d a^{3}}-\frac{9 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{4 d a \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{8 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{d a^{2} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{6 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{d a^{3} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{27 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{4 d a \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{51 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{2 d a^{2} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{38 b^{3} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d a^{3} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{20 b^{4} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d a^{4} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{27 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{4 d a \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{20 b^{2} \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d a^{2} \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2} (a+b)}$$

$$-\frac{14 b^3 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d a^3 \left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2 (a+b)}$$

$$-\frac{9 b}{4 d a \left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2 (a+b)}$$

$$-\frac{5 b^2}{2 d a^2 \left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2 (a+b)}-\frac{15 b \operatorname{arctanh}\left(\frac{2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2 a+4 b}{4 \sqrt{b a+b^2}}\right)}{8 d a^2 (a+b) \sqrt{b a+b^2}}$$

$$-\frac{5 b^2 \operatorname{arctanh}\left(\frac{2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2 a+4 b}{4 \sqrt{b a+b^2}}\right)}{d a^3 (a+b) \sqrt{b a+b^2}}-\frac{3 b^3 \operatorname{arctanh}\left(\frac{2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2 a+4 b}{4 \sqrt{b a+b^2}}\right)}{d a^4 (a+b) \sqrt{b a+b^2}}-\frac{1}{8 d a^3 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$$

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2 d a^3}-\frac{3 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) b}{d a^4}$$

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Problem 23: Result more than twice size of optimal antiderivative.
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$$\int \operatorname{csch}(dx+c)^2 \left(a+b \tanh(dx+c)^3\right)^3 dx$$

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Optimal(type 3, 65 leaves, 3 steps):
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$$-\frac{a^3 \coth(dx+c)}{d}+\frac{3 a^2 b \tanh(dx+c)^2}{2 d}+\frac{3 a b^2 \tanh(dx+c)^5}{5 d}+\frac{b^3 \tanh(dx+c)^8}{8 d}$$

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Result(type 3, 222 leaves):
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$$\frac{1}{d}\left(-\coth(dx+c) a^3+\frac{3 a^2 b \sinh(dx+c)^2}{2 \cosh(dx+c)^2}+3 a b^2\left(-\frac{\sinh(dx+c)^3}{2 \cosh(dx+c)^5}-\frac{3 \sinh(dx+c)}{8 \cosh(dx+c)^5}\right.\right.$$
$$\left.+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4 \operatorname{sech}(dx+c)^2}{15}\right) \tanh(dx+c)}{8}\right)+b^3\left(-\frac{\sinh(dx+c)^6}{2 \cosh(dx+c)^8}-\frac{3 \sinh(dx+c)^4}{4 \cosh(dx+c)^8}-\frac{3 \sinh(dx+c)^2}{8 \cosh(dx+c)^8}\right.$$
$$\left.\left.+\frac{\sinh(dx+c)^2}{8 \cosh(dx+c)^6}+\frac{\sinh(dx+c)^2}{8 \cosh(dx+c)^4}+\frac{\sinh(dx+c)^2}{8 \cosh(dx+c)^2}\right)\right)$$

Problem 24: Result is not expressed in closed-form.

$$\int \frac{\operatorname{csch}(dx+c)^4}{a+b\tanh(dx+c)^3}\,dx$$

Optimal(type 3, 174 leaves, 12 steps):

$$\frac{\coth(dx+c)}{a d}-\frac{\coth(dx+c)^3}{3 a d}-\frac{b\ln(\tanh(dx+c))}{a^2 d}-\frac{b^{1/3}\ln(a^{1/3}+b^{1/3}\tanh(dx+c))}{3 a^{4/3} d}+\frac{b^{1/3}\ln(a^{2/3}-a^{1/3}b^{1/3}\tanh(dx+c)+b^{2/3}\tanh(dx+c)^2)}{6 a^{4/3} d}+\frac{b\ln(a+b\tanh(dx+c)^3)}{3 a^2 d}-\frac{b^{1/3}\arctan\left(\frac{(a^{1/3}-2b^{1/3}\tanh(dx+c))\sqrt{3}}{3 a^{1/3}}\right)\sqrt{3}}{3 a^{4/3} d}$$

Result(type 7, 186 leaves):

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24 d a}+\frac{3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8 d a}-\frac{1}{24 d a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8 d a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d a^2}+\frac{b\left(\sum_{\_R=RootOf(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a)}\frac{(\_R^5 a+4\_R^2 b+3\_R a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5 a+2\_R^3 a+4\_R^2 b+\_R a}\right)}{3 d a^2}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \cosh(dx+c)^2\,(a+b\tanh(dx+c)^2)^2\,dx$$

Optimal(type 3, 47 leaves, 5 steps):

$$\frac{(a-3b)(a+b)x}{2}+\frac{(a+b)^2\cosh(dx+c)\sinh(dx+c)}{2d}+\frac{b^2\tanh(dx+c)}{d}$$

Result(type 3, 95 leaves):

$$\frac{1}{d}\left(a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ba\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)\right)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \cosh(dx+c)\,(a+b\tanh(dx+c)^2)^2\,dx$$

Optimal(type 3, 56 leaves, 5 steps):

$$-\frac{b\,(4a+3b)\arctan(\sinh(dx+c))}{2d}+\frac{(a+b)^2\sinh(dx+c)}{d}+\frac{b^2\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}$$

Result(type 3, 121 leaves):

$$\frac{\sinh(dx+c)\,a^2}{d}+\frac{2\sinh(dx+c)\,a\,b}{d}-\frac{4\,b\,a\arctan\left(e^{dx+c}\right)}{d}+\frac{b^2\sinh(dx+c)^3}{d\cosh(dx+c)^2}+\frac{3\,b^2\sinh(dx+c)}{d\cosh(dx+c)^2}-\frac{3\,b^2\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}-\frac{3\,b^2\arctan\left(e^{dx+c}\right)}{d}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int\cosh(dx+c)\left(a+b\tanh(dx+c)^2\right)^3\,dx$$

Optimal(type 3, 93 leaves, 6 steps):

$$-\frac{3\,b\left(4\,(a+b)^2+(2a+b)^2\right)\arctan(\sinh(dx+c))}{8d}+\frac{(a+b)^3\sinh(dx+c)}{d}+\frac{3\,b^2\,(4a+3b)\operatorname{sech}(dx+c)\tanh(dx+c)}{8d}-\frac{b^3\operatorname{sech}(dx+c)^3\tanh(dx+c)}{4d}$$

Result(type 3, 256 leaves):

$$\frac{\sinh(dx+c)\,a^3}{d}+\frac{3\,a^2\,b\sinh(dx+c)}{d}-\frac{6\,a^2\,b\arctan\left(e^{dx+c}\right)}{d}+\frac{3\,a\,b^2\sinh(dx+c)^3}{d\cosh(dx+c)^2}+\frac{9\,a\,b^2\sinh(dx+c)}{d\cosh(dx+c)^2}-\frac{9\,a\,b^2\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}-\frac{9\,a\,b^2\arctan\left(e^{dx+c}\right)}{d}+\frac{b^3\sinh(dx+c)^5}{d\cosh(dx+c)^4}+\frac{5\,b^3\sinh(dx+c)^3}{d\cosh(dx+c)^4}+\frac{5\,b^3\sinh(dx+c)}{d\cosh(dx+c)^4}-\frac{5\,b^3\operatorname{sech}(dx+c)^3\tanh(dx+c)}{4d}-\frac{15\,b^3\operatorname{sech}(dx+c)\tanh(dx+c)}{8d}-\frac{15\,b^3\arctan\left(e^{dx+c}\right)}{4d}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int\operatorname{sech}(dx+c)\left(a+b\tanh(dx+c)^2\right)^3\,dx$$

Optimal(type 3, 141 leaves, 5 steps):

$$\frac{(2a+b)\left(8a^2+8ba+5b^2\right)\arctan(\sinh(dx+c))}{16d}-\frac{b\left(44a^2+44ba+15b^2\right)\operatorname{sech}(dx+c)\tanh(dx+c)}{48d}-\frac{5\,b\,(2a+b)\operatorname{sech}(dx+c)^3\left(a+(a+b)\sinh(dx+c)^2\right)\tanh(dx+c)}{24d}-\frac{b\operatorname{sech}(dx+c)^5\left(a+(a+b)\sinh(dx+c)^2\right)^2\tanh(dx+c)}{6d}$$

Result(type 3, 333 leaves):

$$\frac{2\,a^3\arctan\left(e^{dx+c}\right)}{d}-\frac{3\,a^2\,b\sinh(dx+c)}{d\cosh(dx+c)^2}+\frac{3\,a^2\,b\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}+\frac{3\,a^2\,b\arctan\left(e^{dx+c}\right)}{d}-\frac{3\,a\,b^2\sinh(dx+c)^3}{d\cosh(dx+c)^4}-\frac{3\,a\,b^2\sinh(dx+c)}{d\cosh(dx+c)^4}+\frac{3\,a\,b^2\tanh(dx+c)\operatorname{sech}(dx+c)^3}{4d}+\frac{9\,a\,b^2\operatorname{sech}(dx+c)\tanh(dx+c)}{8d}+\frac{9\,a\,b^2\arctan\left(e^{dx+c}\right)}{4d}-\frac{b^3\sinh(dx+c)^5}{d\cosh(dx+c)^6}-\frac{5\,b^3\sinh(dx+c)^3}{3\,d\cosh(dx+c)^6}$$

$$-\frac{b^3 \sinh(dx+c)}{d \cosh(dx+c)^6} + \frac{b^3 \tanh(dx+c)\,\mathrm{sech}(dx+c)^5}{6d} + \frac{5 b^3 \,\mathrm{sech}(dx+c)^3 \tanh(dx+c)}{24d} + \frac{5 b^3 \,\mathrm{sech}(dx+c) \tanh(dx+c)}{16d} + \frac{5 b^3 \arctan\left(e^{dx+c}\right)}{8d}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \mathrm{sech}(dx+c)^2 \left(a + b \tanh(dx+c)^2\right)^3 \mathrm{d}x$$

Optimal(type 3, 63 leaves, 3 steps):

$$\frac{a^3 \tanh(dx+c)}{d} + \frac{a^2 b \tanh(dx+c)^3}{d} + \frac{3 a b^2 \tanh(dx+c)^5}{5d} + \frac{b^3 \tanh(dx+c)^7}{7d}$$

Result(type 3, 226 leaves):

$$\frac{1}{d}\left(\tanh(dx+c)\, a^3 + 3 a^2 b \left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3} + \frac{\left(\frac{2}{3} + \frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right) + 3 a b^2 \left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5} - \frac{3\sinh(dx+c)}{8\cosh(dx+c)^5} + \frac{3\left(\frac{8}{15} + \frac{\mathrm{sech}(dx+c)^4}{5} + \frac{4\,\mathrm{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right) + b^3 \left(-\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^7} - \frac{5\sinh(dx+c)^3}{8\cosh(dx+c)^7} - \frac{5\sinh(dx+c)}{16\cosh(dx+c)^7} + \frac{5\left(\frac{16}{35} + \frac{\mathrm{sech}(dx+c)^6}{7} + \frac{6\,\mathrm{sech}(dx+c)^4}{35} + \frac{8\,\mathrm{sech}(dx+c)^2}{35}\right)\tanh(dx+c)}{16}\right)\right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{sech}(dx+c)^5}{a + b\tanh(dx+c)^2} \mathrm{d}x$$

Optimal(type 3, 74 leaves, 5 steps):

$$-\frac{(2a+3b)\arctan(\sinh(dx+c))}{2b^2 d} + \frac{(a+b)^{3/2} \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{b^2 d\sqrt{a}} - \frac{\mathrm{sech}(dx+c)\tanh(dx+c)}{2bd}$$

Result(type 3, 835 leaves):

$$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2} - \frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2} - \frac{3\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db} - \frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{db^2}$$

$$+\frac{a^2\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{d\,b^2\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}+\frac{2a\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{d\,b\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}+\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{d\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}$$

$$+\frac{a^2\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{d\,b\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}+\frac{2a\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{d\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}+\frac{b\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{d\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}$$

$$-\frac{a^2\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{d\,b^2\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}-\frac{2a\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{d\,b\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}-\frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{d\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}$$

$$+\frac{a^2\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{d\,b\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{2a\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{d\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{b\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{d\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int\frac{\cosh(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^2}\,dx$$

Optimal(type 3, 89 leaves, 5 steps):

$$\frac{b(4a+b)\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}d}+\frac{\sinh(dx+c)}{(a+b)^2d}+\frac{b^2\sinh(dx+c)}{2a(a+b)^2d\left(a+(a+b)\sinh(dx+c)^2\right)}$$

Result(type 3, 800 leaves):

$$-\frac{1}{d(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d(a+b)^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}$$

$$+\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d(a+b)^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}$$

$$+\frac{2 b^{2} a \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{-a^{2}-2 b a+2 \sqrt{a^{2} b (a+b)}}}\right)}{d (a+b)^{2} \sqrt{a^{2} b (a+b)} \sqrt{-a^{2}-2 b a+2 \sqrt{a^{2} b (a+b)}}}-\frac{2 b \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{-a^{2}-2 b a+2 \sqrt{a^{2} b (a+b)}}}\right)}{d (a+b)^{2} \sqrt{-a^{2}-2 b a+2 \sqrt{a^{2} b (a+b)}}}$$

$$+\frac{2 b^{2} a \arctan\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{a^{2}+2 b a+2 \sqrt{a^{2} b (a+b)}}}\right)}{d (a+b)^{2} \sqrt{a^{2} b (a+b)} \sqrt{a^{2}+2 b a+2 \sqrt{a^{2} b (a+b)}}}+\frac{2 b \arctan\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{a^{2}+2 b a+2 \sqrt{a^{2} b (a+b)}}}\right)}{d (a+b)^{2} \sqrt{a^{2}+2 b a+2 \sqrt{a^{2} b (a+b)}}}$$

$$+\frac{b^{3} \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{-a^{2}-2 b a+2 \sqrt{a^{2} b (a+b)}}}\right)}{2 d (a+b)^{2} \sqrt{a^{2} b (a+b)} \sqrt{-a^{2}-2 b a+2 \sqrt{a^{2} b (a+b)}}}-\frac{b^{2} \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{-a^{2}-2 b a+2 \sqrt{a^{2} b (a+b)}}}\right)}{2 d (a+b)^{2} a \sqrt{-a^{2}-2 b a+2 \sqrt{a^{2} b (a+b)}}}$$

$$+\frac{b^{3} \arctan\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{a^{2}+2 b a+2 \sqrt{a^{2} b (a+b)}}}\right)}{2 d (a+b)^{2} \sqrt{a^{2} b (a+b)} \sqrt{a^{2}+2 b a+2 \sqrt{a^{2} b (a+b)}}}+\frac{b^{2} \arctan\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a}{\sqrt{a^{2}+2 b a+2 \sqrt{a^{2} b (a+b)}}}\right)}{2 d (a+b)^{2} a \sqrt{a^{2}+2 b a+2 \sqrt{a^{2} b (a+b)}}}$$

$$-\frac{1}{d (a+b)^{2} \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(d x+c)}{\left(a+b \tanh(d x+c)^{2}\right)^{2}}\, dx$$

Optimal(type 3, 71 leaves, 3 steps):

$$\frac{(2 a+b) \arctan\left(\frac{\sinh(d x+c) \sqrt{a+b}}{\sqrt{a}}\right)}{2 a^{3/2} (a+b)^{3/2} d}+\frac{b \sinh(d x+c)}{2 a (a+b) d \left(a+(a+b) \sinh(d x+c)^{2}\right)}$$

Result(type 3, 1800 leaves):

$$-\frac{b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right) (a+b) a}$$

$$+\frac{b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{d\left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{4}+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+a\right)(a+b) a}$$

$$+\frac{a^{3} \arctan\left(\frac{\left(a^{3}+a^{2} b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}\right) b}{d \sqrt{a^{2} b (a+b)^{3}}\,(a+b) \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}$$

$$+\frac{5 a^{2} \arctan\left(\frac{\left(a^{3}+a^{2} b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}\right) b^{2}}{2 d \sqrt{a^{2} b (a+b)^{3}}\,(a+b) \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}$$

$$+\frac{2 a \arctan\left(\frac{\left(a^{3}+a^{2} b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}\right) b^{3}}{d \sqrt{a^{2} b (a+b)^{3}}\,(a+b) \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}$$

$$+\frac{\arctan\left(\frac{\left(a^{3}+a^{2} b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}\right) b^{4}}{2 d \sqrt{a^{2} b (a+b)^{3}}\,(a+b) \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}$$

$$+\frac{a \arctan\left(\frac{\left(a^{3}+a^{2} b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}\right)}{d (a+b) \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}+\frac{3 \arctan\left(\frac{\left(a^{3}+a^{2} b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}\right) b}{2 d (a+b) \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}$$

$$+\frac{\arctan\left(\frac{\left(a^{3}+a^{2} b\right) \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}\right) b^{2}}{2 d a (a+b) \sqrt{(a+b)\left(a^{3}+3 a^{2} b+2 a b^{2}+2 \sqrt{a^{2} b (a+b)^{3}}\right)}}$$

$$-\frac{a^3 \operatorname{arctanh}\left(\frac{(-a^3-a^2 b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}\right) b}{d\sqrt{a^2 b (a+b)^3}\,(a+b)\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}$$

$$-\frac{5 a^2 \operatorname{arctanh}\left(\frac{(-a^3-a^2 b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}\right) b^2}{2 d\sqrt{a^2 b (a+b)^3}\,(a+b)\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}$$

$$-\frac{2 a \operatorname{arctanh}\left(\frac{(-a^3-a^2 b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}\right) b^3}{d\sqrt{a^2 b (a+b)^3}\,(a+b)\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{(-a^3-a^2 b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}\right) b^4}{2 d\sqrt{a^2 b (a+b)^3}\,(a+b)\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}$$

$$+\frac{a \operatorname{arctanh}\left(\frac{(-a^3-a^2 b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}\right)}{d (a+b)\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}+\frac{3 \operatorname{arctanh}\left(\frac{(-a^3-a^2 b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}\right) b}{2 d (a+b)\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}$$

$$+\frac{\operatorname{arctanh}\left(\frac{(-a^3-a^2 b)\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}{a\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}\right) b^2}{2 d a (a+b)\sqrt{\left(-a^3-3 a^2 b-2 a b^2+2\sqrt{a^2 b (a+b)^3}\right)(a+b)}}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(d x+c)^2}{\left(a+b\tanh(d x+c)^2\right)^2}\, dx$$

Optimal(type 3, 54 leaves, 3 steps):

$$\frac{\arctan\left(\frac{\sqrt{b}\,\tanh(dx+c)}{\sqrt{a}}\right)}{2\,a^{3/2}\,d\sqrt{b}}+\frac{\tanh(dx+c)}{2\,a\,d\,\left(a+b\tanh(dx+c)^2\right)}$$

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Result(type 3, 553 leaves):
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$$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}$$

$$+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}-\frac{a\,\mathrm{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2b\,(a+b)}}}\right)}{2\,d\sqrt{a^2b\,(a+b)}\sqrt{-a^2-2\,b\,a+2\sqrt{a^2b\,(a+b)}}}$$

$$-\frac{\mathrm{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2b\,(a+b)}}}\right)b}{2\,d\sqrt{a^2b\,(a+b)}\sqrt{-a^2-2\,b\,a+2\sqrt{a^2b\,(a+b)}}}+\frac{\mathrm{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2b\,(a+b)}}}\right)}{2\,d\,a\sqrt{-a^2-2\,b\,a+2\sqrt{a^2b\,(a+b)}}}$$

$$-\frac{a\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2b\,(a+b)}}}\right)}{2\,d\sqrt{a^2b\,(a+b)}\sqrt{a^2+2\,b\,a+2\sqrt{a^2b\,(a+b)}}}-\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2b\,(a+b)}}}\right)b}{2\,d\sqrt{a^2b\,(a+b)}\sqrt{a^2+2\,b\,a+2\sqrt{a^2b\,(a+b)}}}$$

$$-\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2b\,(a+b)}}}\right)}{2\,d\,a\sqrt{a^2+2\,b\,a+2\sqrt{a^2b\,(a+b)}}}$$

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Problem 35: Result more than twice size of optimal antiderivative.
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$$\int\frac{\mathrm{sech}(dx+c)^3}{\left(a+b\tanh(dx+c)^2\right)^2}\,dx$$

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Optimal(type 3, 60 leaves, 3 steps):
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$$\frac{\sinh(dx+c)}{2ad\left(a+(a+b)\sinh(dx+c)^2\right)}+\frac{\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a+b}}$$

Result(type 3, 406 leaves):

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}$$

$$+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}+\frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)b}{2d\sqrt{a^2b(a+b)}\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)}{2da\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}+\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)b}{2d\sqrt{a^2b(a+b)}\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}+\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)}{2da\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int\frac{\operatorname{sech}(dx+c)^5}{\left(a+b\tanh(dx+c)^2\right)^2}\,dx$$

Optimal(type 3, 90 leaves, 5 steps):

$$\frac{\arctan(\sinh(dx+c))}{b^2d}+\frac{(a+b)\sinh(dx+c)}{2abd\left(a+(a+b)\sinh(dx+c)^2\right)}-\frac{(2a-b)\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)\sqrt{a+b}}{2a^{3/2}b^2d}$$

Result(type 3, 1120 leaves):

$$\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db^2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{db\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}$$

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}$$

$$+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}$$

$$+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\,b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}-\frac{a^2\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{d\,b\sqrt{a^2 b\,(a+b)}\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}$$

$$-\frac{a\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{2\,d\sqrt{a^2 b\,(a+b)}\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}+\frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)b}{2\,d\sqrt{a^2 b\,(a+b)}\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}$$

$$+\frac{a\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{d\,b^2\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}+\frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{2\,d\,b\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}-\frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{2\,d\,a\sqrt{-a^2-2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}$$

$$-\frac{a^2\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{d\,b\sqrt{a^2 b\,(a+b)}\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}-\frac{a\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{2\,d\sqrt{a^2 b\,(a+b)}\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}$$

$$+\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)b}{2\,d\sqrt{a^2 b\,(a+b)}\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}-\frac{a\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{d\,b^2\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}-\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{2\,d\,b\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}$$

$$+\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}\right)}{2\,d\,a\sqrt{a^2+2\,b\,a+2\sqrt{a^2 b\,(a+b)}}}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(dx+c)^7}{\left(a+b\tanh(dx+c)^2\right)^2}\, dx$$

Optimal(type 3, 139 leaves, 6 steps):

$$\frac{(4a+5b)\arctan(\sinh(dx+c))}{2b^3 d} - \frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{2a^{3/2}b^3 d} + \frac{(a+b)(2a+b)\sinh(dx+c)}{2ab^2 d\left(a+(a+b)\sinh(dx+c)^2\right)} - \frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2bd\left(a+(a+b)\sinh(dx+c)^2\right)}$$

Result(type 3, 1628 leaves):

$$-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{db^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)} + \frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{db^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)} + \frac{2a^2\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)}{db^3\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}$$

$$-\frac{2a^2\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)}{db^3\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}} + \frac{7a\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)}{2db^2\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}} - \frac{7a\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)}{2db^2\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}$$

$$-\frac{a\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)}{d\sqrt{a^2b(a+b)}\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}} + \frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)b}{2d\sqrt{a^2b(a+b)}\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}$$

$$-\frac{a\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)}{d\sqrt{a^2b(a+b)}\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}} + \frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)b}{2d\sqrt{a^2b(a+b)}\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}$$

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}$$

$$+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)a}-\frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)}{2da\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}$$

$$+\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)}{2da\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}+\frac{5\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db^2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{db^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{db^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}$$

$$+\frac{4\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{db^3}-\frac{2a^3\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)}{db^2\sqrt{a^2b(a+b)}\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}-\frac{2a^3\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)}{db^2\sqrt{a^2b(a+b)}\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}$$

$$-\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{db\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}$$

$$+\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}+\frac{\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)}{db\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}$$

$$-\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)}{db\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}-\frac{7a^2\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}\right)}{2db\sqrt{a^2b(a+b)}\sqrt{-a^2-2ba+2\sqrt{a^2b(a+b)}}}$$

$$-\frac{7a^2\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}\right)}{2db\sqrt{a^2b(a+b)}\sqrt{a^2+2ba+2\sqrt{a^2b(a+b)}}}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^3}\,dx$$

Optimal(type 3, 140 leaves, 6 steps):

$$\frac{3b\left(8a^2+4ba+b^2\right)\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d}+\frac{\sinh(dx+c)}{(a+b)^3 d}+\frac{b^3\sinh(dx+c)}{4a(a+b)^3 d\left(a+(a+b)\sinh(dx+c)^2\right)^2}$$
$$+\frac{3b^2(4a+b)\sinh(dx+c)}{8a^2(a+b)^3 d\left(a+(a+b)\sinh(dx+c)^2\right)}$$

Result(type 3, 1801 leaves):

$$-\frac{1}{d(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$
$$-\frac{5b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2a}$$
$$-\frac{3b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$
$$-\frac{45b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2a}$$
$$-\frac{3b^4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2a^2}$$
$$+\frac{3b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$

$$+\frac{45\,b^{3}\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{3}}{4\,d\,(a+b)^{3}\left(a\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{4}+2\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{2}a+4\,b\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}a}$$

$$+\frac{3\,b^{4}\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{3}}{d\,(a+b)^{3}\left(a\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{4}+2\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{2}a+4\,b\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}a^{2}}$$

$$+\frac{3\,b^{2}\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)}{d\,(a+b)^{3}\left(a\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{4}+2\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{2}a+4\,b\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}$$

$$+\frac{5\,b^{3}\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)}{4\,d\,(a+b)^{3}\left(a\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{4}+2\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{2}a+4\,b\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}a}$$

$$+\frac{3\,b^{2}a^{3}\arctan\left(\frac{\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}\right)}{d\,(a+b)^{3}\sqrt{a^{4}b\,(a+b)}\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}+\frac{3\,b^{3}a^{2}\arctan\left(\frac{\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}\right)}{2\,d\,(a+b)^{3}\sqrt{a^{4}b\,(a+b)}\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}$$

$$+\frac{3\,b^{4}a\arctan\left(\frac{\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}\right)}{8\,d\,(a+b)^{3}\sqrt{a^{4}b\,(a+b)}\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}+\frac{3\,b\,a\arctan\left(\frac{\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}\right)}{d\,(a+b)^{3}\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}$$

$$+\frac{3\,b^{2}\arctan\left(\frac{\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}\right)}{2\,d\,(a+b)^{3}\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}+\frac{3\,b^{3}\arctan\left(\frac{\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}\right)}{8\,d\,(a+b)^{3}a\sqrt{\left(a^{3}+2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}$$

$$+\frac{3\,b^{2}a^{3}\operatorname{arctanh}\left(\frac{\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(-a^{3}-2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}\right)}{d\,(a+b)^{3}\sqrt{a^{4}b\,(a+b)}\sqrt{\left(-a^{3}-2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}+\frac{3\,b^{3}a^{2}\operatorname{arctanh}\left(\frac{\tanh\left(\frac{d\,x}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(-a^{3}-2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}\right)}{2\,d\,(a+b)^{3}\sqrt{a^{4}b\,(a+b)}\sqrt{\left(-a^{3}-2\,a^{2}b+2\sqrt{a^{4}b\,(a+b)}\right)a}}$$

$$+\frac{3 b^4 a \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a^2}{\sqrt{\left(-a^3-2 a^2 b+2 \sqrt{a^4 b (a+b)}\right) a}}\right)}{8 d (a+b)^3 \sqrt{a^4 b (a+b)} \sqrt{\left(-a^3-2 a^2 b+2 \sqrt{a^4 b (a+b)}\right) a}}-\frac{3 b a \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a^2}{\sqrt{\left(-a^3-2 a^2 b+2 \sqrt{a^4 b (a+b)}\right) a}}\right)}{d (a+b)^3 \sqrt{\left(-a^3-2 a^2 b+2 \sqrt{a^4 b (a+b)}\right) a}}$$

$$-\frac{3 b^2 \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a^2}{\sqrt{\left(-a^3-2 a^2 b+2 \sqrt{a^4 b (a+b)}\right) a}}\right)}{2 d (a+b)^3 \sqrt{\left(-a^3-2 a^2 b+2 \sqrt{a^4 b (a+b)}\right) a}}-\frac{3 b^3 \operatorname{arctanh}\left(\frac{\tanh\left(\frac{d x}{2}+\frac{c}{2}\right) a^2}{\sqrt{\left(-a^3-2 a^2 b+2 \sqrt{a^4 b (a+b)}\right) a}}\right)}{8 d (a+b)^3 a \sqrt{\left(-a^3-2 a^2 b+2 \sqrt{a^4 b (a+b)}\right) a}}$$

$$-\frac{1}{d (a+b)^3 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(d x+c)^5}{\left(a+b \tanh(d x+c)^2\right)^3} \, dx$$

Optimal(type 3, 90 leaves, 4 steps):

$$\frac{\sinh(d x+c)}{4 a d \left(a+(a+b) \sinh(d x+c)^2\right)^2}+\frac{3 \sinh(d x+c)}{8 a^2 d \left(a+(a+b) \sinh(d x+c)^2\right)}+\frac{3 \arctan\left(\frac{\sinh(d x+c) \sqrt{a+b}}{\sqrt{a}}\right)}{8 a^{5/2} d \sqrt{a+b}}$$

Result(type 3, 707 leaves):

$$-\frac{5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^7}{4 d \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+a\right)^2 a}$$

$$+\frac{3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^5}{4 d \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+a\right)^2 a}$$

$$-\frac{3 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^5}{d \left(a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^2+a\right)^2 a^2}$$

$$-\frac{3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}a}$$

$$+\frac{3b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}a^{2}}$$

$$+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}a}+\frac{3a\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(a^{3}+2a^{2}b+2\sqrt{a^{4}b(a+b)}\right)a}}\right)b}{8d\sqrt{a^{4}b(a+b)}\sqrt{\left(a^{3}+2a^{2}b+2\sqrt{a^{4}b(a+b)}\right)a}}$$

$$+\frac{3\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(a^{3}+2a^{2}b+2\sqrt{a^{4}b(a+b)}\right)a}}\right)}{8da\sqrt{\left(a^{3}+2a^{2}b+2\sqrt{a^{4}b(a+b)}\right)a}}+\frac{3a\,\mathrm{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(-a^{3}-2a^{2}b+2\sqrt{a^{4}b(a+b)}\right)a}}\right)b}{8d\sqrt{a^{4}b(a+b)}\sqrt{\left(-a^{3}-2a^{2}b+2\sqrt{a^{4}b(a+b)}\right)a}}$$

$$-\frac{3\,\mathrm{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}}{\sqrt{\left(-a^{3}-2a^{2}b+2\sqrt{a^{4}b(a+b)}\right)a}}\right)}{8da\sqrt{\left(-a^{3}-2a^{2}b+2\sqrt{a^{4}b(a+b)}\right)a}}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{sech}(dx+c)^{7}}{\left(a+b\tanh(dx+c)^{2}\right)^{3}}\,dx$$

Optimal(type 3, 142 leaves, 6 steps):

$$-\frac{\arctan(\sinh(dx+c))}{b^{3}d}+\frac{(a+b)\sinh(dx+c)}{4abd\left(a+(a+b)\sinh(dx+c)^{2}\right)^{2}}-\frac{(4a-3b)(a+b)\sinh(dx+c)}{8a^{2}b^{2}d\left(a+(a+b)\sinh(dx+c)^{2}\right)}$$

$$+\frac{\left(8a^{2}-4ba+3b^{2}\right)\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)\sqrt{a+b}}{8a^{5/2}b^{3}d}$$

Result(type ?, 2224 leaves): Display of huge result suppressed!

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \tanh(dx+c)\,\left(a+b\tanh(dx+c)^2\right)^2 \,dx$$

Optimal(type 3, 53 leaves, 4 steps):

$$\frac{(a+b)^2\ln(\cosh(dx+c))}{d}-\frac{b\,(a+b)\,\tanh(dx+c)^2}{2\,d}-\frac{\left(a+b\tanh(dx+c)^2\right)^2}{4\,d}$$

Result(type 3, 148 leaves):

$$-\frac{\tanh(dx+c)^4 b^2}{4\,d}-\frac{a\,b\tanh(dx+c)^2}{d}-\frac{\tanh(dx+c)^2 b^2}{2\,d}-\frac{\ln(\tanh(dx+c)-1)\,a^2}{2\,d}-\frac{\ln(\tanh(dx+c)-1)\,b\,a}{d}-\frac{\ln(\tanh(dx+c)-1)\,b^2}{2\,d}-\frac{\ln(\tanh(dx+c)+1)\,a^2}{2\,d}-\frac{\ln(\tanh(dx+c)+1)\,b\,a}{d}-\frac{\ln(\tanh(dx+c)+1)\,b^2}{2\,d}$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\tanh(dx+c)^2\right)^2 \,dx$$

Optimal(type 3, 41 leaves, 4 steps):

$$(a+b)^2 x-\frac{b\,(2\,a+b)\,\tanh(dx+c)}{d}-\frac{b^2\tanh(dx+c)^3}{3\,d}$$

Result(type 3, 143 leaves):

$$-\frac{b^2\tanh(dx+c)^3}{3\,d}-\frac{2\tanh(dx+c)\,a\,b}{d}-\frac{b^2\tanh(dx+c)}{d}-\frac{\ln(\tanh(dx+c)-1)\,a^2}{2\,d}-\frac{\ln(\tanh(dx+c)-1)\,b\,a}{d}-\frac{\ln(\tanh(dx+c)-1)\,b^2}{2\,d}+\frac{\ln(\tanh(dx+c)+1)\,a^2}{2\,d}+\frac{\ln(\tanh(dx+c)+1)\,b\,a}{d}+\frac{\ln(\tanh(dx+c)+1)\,b^2}{2\,d}$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \tanh(dx+c)^4\left(a+b\tanh(dx+c)^2\right)^3 \,dx$$

Optimal(type 3, 106 leaves, 4 steps):

$$(a+b)^3 x-\frac{(a+b)^3\tanh(dx+c)}{d}-\frac{(a+b)^3\tanh(dx+c)^3}{3\,d}-\frac{b\left(3\,a^2+3\,b\,a+b^2\right)\tanh(dx+c)^5}{5\,d}-\frac{b^2\,(3\,a+b)\,\tanh(dx+c)^7}{7\,d}-\frac{b^3\tanh(dx+c)^9}{9\,d}$$

Result(type 3, 364 leaves):

$$-\frac{3\,a\,b^2\tanh(dx+c)}{d}-\frac{3\,a^2\,b\tanh(dx+c)}{d}-\frac{a^2\,b\tanh(dx+c)^3}{d}-\frac{3\tanh(dx+c)^7\,a\,b^2}{7\,d}-\frac{3\tanh(dx+c)^5\,a^2\,b}{5\,d}-\frac{3\,a\,b^2\tanh(dx+c)^5}{5\,d}-\frac{a^3\tanh(dx+c)}{d}-\frac{\tanh(dx+c)\,b^3}{d}-\frac{b^3\tanh(dx+c)^3}{3\,d}-\frac{\ln(\tanh(dx+c)-1)\,a^3}{2\,d}-\frac{3\ln(\tanh(dx+c)-1)\,a^2\,b}{2\,d}-\frac{3\ln(\tanh(dx+c)-1)\,a\,b^2}{2\,d}-\frac{\ln(\tanh(dx+c)-1)\,b^3}{2\,d}-\frac{\tanh(dx+c)^3\,a^3}{3\,d}-\frac{b^3\tanh(dx+c)^7}{7\,d}-\frac{b^3\tanh(dx+c)^5}{5\,d}+\frac{\ln(\tanh(dx+c)+1)\,a^3}{2\,d}+\frac{3\ln(\tanh(dx+c)+1)\,a^2\,b}{2\,d}+\frac{3\ln(\tanh(dx+c)+1)\,a\,b^2}{2\,d}+\frac{\ln(\tanh(dx+c)+1)\,b^3}{2\,d}-\frac{a\,b^2\tanh(dx+c)^3}{d}-\frac{b^3\tanh(dx+c)^9}{9\,d}$$

Problem 49: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(dx+c)^3}{\left(a+b\tanh(dx+c)^2\right)^2}\, dx$$

Optimal(type 3, 118 leaves, 4 steps):

$$-\frac{\coth(dx+c)^2}{2a^2 d}+\frac{\ln(\cosh(dx+c))}{(a+b)^2 d}+\frac{(a-2b)\ln(\tanh(dx+c))}{a^3 d}+\frac{b^2(3a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{2a^3(a+b)^2 d}-\frac{b^2}{2a^2(a+b)d\left(a+b\tanh(dx+c)^2\right)}$$

Result(type 3, 382 leaves):

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8da^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d(a+b)^2}+\frac{2b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da^2(a+b)^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}$$

$$+\frac{2b^4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{da^3(a+b)^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}$$

$$+\frac{3b^2\ln\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}{2da^2(a+b)^2}$$

$$+\frac{b^3\ln\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}{da^3(a+b)^2}-\frac{1}{8da^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da^2}$$

$$-\frac{2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{da^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a+b)^2}$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(dx+c)^6}{\left(a+b\tanh(dx+c)^2\right)^3}\, dx$$

Optimal(type 3, 130 leaves, 6 steps):

$$\frac{x}{(a+b)^3}-\frac{(3a^2+10ba+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(dx+c)}{\sqrt{a}}\right)\sqrt{a}}{8b^{5/2}(a+b)^3 d}+\frac{a\tanh(dx+c)^3}{4b(a+b)d\left(a+b\tanh(dx+c)^2\right)^2}+\frac{a(3a+7b)\tanh(dx+c)}{8b^2(a+b)^2 d\left(a+b\tanh(dx+c)^2\right)}$$

Result(type 3, 351 leaves):

$$-\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^3}+\frac{\ln(\tanh(dx+c)+1)}{2d(a+b)^3}+\frac{5a^3\tanh(dx+c)^3}{8d(a+b)^3\left(a+b\tanh(dx+c)^2\right)^2 b}+\frac{7a^2\tanh(dx+c)^3}{4d(a+b)^3\left(a+b\tanh(dx+c)^2\right)^2}$$

$$+\frac{9ab\tanh(dx+c)^3}{8d(a+b)^3(a+b\tanh(dx+c)^2)^2}+\frac{3a^4\tanh(dx+c)}{8d(a+b)^3(a+b\tanh(dx+c)^2)^2b^2}+\frac{5a^3\tanh(dx+c)}{4d(a+b)^3(a+b\tanh(dx+c)^2)^2b}$$

$$+\frac{7a^2\tanh(dx+c)}{8d(a+b)^3(a+b\tanh(dx+c)^2)^2}-\frac{3a^3\arctan\left(\frac{\tanh(dx+c)b}{\sqrt{ba}}\right)}{8d(a+b)^3b^2\sqrt{ba}}-\frac{5a^2\arctan\left(\frac{\tanh(dx+c)b}{\sqrt{ba}}\right)}{4d(a+b)^3b\sqrt{ba}}-\frac{15a\arctan\left(\frac{\tanh(dx+c)b}{\sqrt{ba}}\right)}{8d(a+b)^3\sqrt{ba}}$$

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Problem 51: Result more than twice size of optimal antiderivative.
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$$\int\frac{\coth(dx+c)}{(a+b\tanh(dx+c)^2)^3}\,dx$$

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Optimal(type 3, 132 leaves, 4 steps):
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$$\frac{\ln(\cosh(dx+c))}{(a+b)^3d}+\frac{\ln(\tanh(dx+c))}{a^3d}-\frac{b(3a^2+3ba+b^2)\ln(a+b\tanh(dx+c)^2)}{2a^3(a+b)^3d}+\frac{b}{4a(a+b)d(a+b\tanh(dx+c)^2)^2}$$

$$+\frac{b(2a+b)}{2a^2(a+b)^2d(a+b\tanh(dx+c)^2)}$$

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Result(type 3, 951 leaves):
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$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d(a+b)^3}-\frac{6b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$

$$-\frac{10b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d(a+b)^3a\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$

$$-\frac{4b^4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d(a+b)^3a^2\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$

$$-\frac{12b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d(a+b)^3\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$

$$-\frac{40b^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d(a+b)^3a\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$

$$-\frac{40 b^4 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d (a+b)^3 a^2 \left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$
$$-\frac{12 b^5 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d (a+b)^3 a^3 \left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$
$$-\frac{6 b^2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d (a+b)^3 \left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$
$$-\frac{10 b^3 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d (a+b)^3 a \left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$
$$-\frac{4 b^4 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d (a+b)^3 a^2 \left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}$$
$$-\frac{3 b \ln\left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}{2 d (a+b)^3 a}$$
$$-\frac{3 b^2 \ln\left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}{2 d (a+b)^3 a^2}$$
$$-\frac{b^3 \ln\left(a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4 b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}{2 d (a+b)^3 a^3}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d a^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d (a+b)^3}$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int \frac{\coth(dx+c)^4}{\left(a+b\tanh(dx+c)^2\right)^3}\, dx$$

Optimal(type 3, 210 leaves, 8 steps):

$$\frac{x}{(a+b)^3}+\frac{b^{5/2}\left(63a^2+90ba+35b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(dx+c)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3 d}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(dx+c)}{8a^4(a+b)^2 d}$$
$$-\frac{\left(8a^2+55ba+35b^2\right)\coth(dx+c)^3}{24a^3(a+b)^2 d}+\frac{b\coth(dx+c)^3}{4a(a+b)d\left(a+b\tanh(dx+c)^2\right)^2}+\frac{b(11a+7b)\coth(dx+c)^3}{8a^2(a+b)^2 d\left(a+b\tanh(dx+c)^2\right)}$$

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Result(type ?, 2138 leaves): Display of huge result suppressed!
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Problem 55: Result more than twice size of optimal antiderivative.
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$$\int\sqrt{a+b\tanh(x)^2}\tanh(x)^5\,dx$$

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Optimal(type 3, 71 leaves, 7 steps):
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$$\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(x)^2}}{\sqrt{a+b}}\right)\sqrt{a+b}-\sqrt{a+b\tanh(x)^2}+\frac{(a-b)\left(a+b\tanh(x)^2\right)^{3/2}}{3b^2}-\frac{\left(a+b\tanh(x)^2\right)^{5/2}}{5b^2}$$

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Result(type 3, 287 leaves):
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$$-\frac{\left(a+b\tanh(x)^2\right)^{3/2}}{3b}-\frac{\tanh(x)^2\left(a+b\tanh(x)^2\right)^{3/2}}{5b}+\frac{2a\left(a+b\tanh(x)^2\right)^{3/2}}{15b^2}-\frac{\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{2}$$
$$-\frac{\sqrt{b}\ln\left(\frac{(\tanh(x)-1)b+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}\right)}{2}$$
$$+\frac{\sqrt{a+b}\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2}$$
$$-\frac{\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{2}+\frac{\sqrt{b}\ln\left(\frac{(1+\tanh(x))b-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}\right)}{2}$$
$$+\frac{\sqrt{a+b}\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2}$$

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Problem 56: Unable to integrate problem.
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$$\int\coth(x)^4\sqrt{a+b\tanh(x)^2}\,dx$$

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Optimal(type 3, 64 leaves, 6 steps):
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$$\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right)\sqrt{a+b}-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh(x)^2}}{3a}-\frac{\coth(x)^3\sqrt{a+b\tanh(x)^2}}{3}$$

Result(type 8, 17 leaves):

$$\int \coth(x)^4 \sqrt{a+b\tanh(x)^2}\, dx$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \tanh(x)^2 \left(a+b\tanh(x)^2\right)^{3/2} dx$$

Optimal(type 3, 101 leaves, 8 steps):

$$(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right) - \frac{(3a^2+12ba+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right)}{8\sqrt{b}} - \frac{(5a+4b)\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{8} - \frac{b\sqrt{a+b\tanh(x)^2}\,\tanh(x)^3}{4}$$

Result(type 3, 632 leaves):

$$-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{3/2}}{4} - \frac{3a\tanh(x)\sqrt{a+b\tanh(x)^2}}{8} - \frac{3a^2\ln\left(\tanh(x)\sqrt{b}+\sqrt{a+b\tanh(x)^2}\right)}{8\sqrt{b}}$$
$$-\frac{\left(b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b\right)^{3/2}}{6} - \frac{b\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}\,\tanh(x)}{4}$$
$$-\frac{3\sqrt{b}\ln\left(\frac{(\tanh(x)-1)b+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}\right)a}{4}$$
$$+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)\sqrt{a+b}\,a}{2}$$
$$-\frac{\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}\,a}{2}$$
$$+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)\sqrt{a+b}\,b}{2}$$
$$-\frac{b^{3/2}\ln\left(\frac{(\tanh(x)-1)b+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}\right)}{2} - \frac{\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}\,b}{2}$$
$$+\frac{\left(b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b\right)^{3/2}}{6} - \frac{b\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}\,\tanh(x)}{4}$$

$$-\frac{3\sqrt{b}\,\ln\left(\frac{(1+\tanh(x))\,b-b}{\sqrt{b}}+\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\right)a}{4}$$

$$-\frac{\ln\left(\frac{2\,a+2\,b-2\,(1+\tanh(x))\,b+2\sqrt{a+b}\,\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}{1+\tanh(x)}\right)\sqrt{a+b}\;a}{2}$$

$$+\frac{\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\;a}{2}-\frac{\ln\left(\frac{(1+\tanh(x))\,b-b}{\sqrt{b}}+\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\right)b^{3/2}}{2}$$

$$-\frac{\ln\left(\frac{2\,a+2\,b-2\,(1+\tanh(x))\,b+2\sqrt{a+b}\,\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}{1+\tanh(x)}\right)\sqrt{a+b}\;b}{2}$$

$$+\frac{\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\;b}{2}$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int\tanh(x)\,\left(a+b\tanh(x)^2\right)^{3/2}\,dx$$

Optimal(type 3, 51 leaves, 6 steps):

$$(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(x)^2}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh(x)^2}-\frac{\left(a+b\tanh(x)^2\right)^{3/2}}{3}$$

Result(type 3, 577 leaves):

$$-\frac{\left(b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b\right)^{3/2}}{6}-\frac{b\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}\;\tanh(x)}{4}$$

$$-\frac{3\sqrt{b}\,\ln\left(\frac{(\tanh(x)-1)\,b+b}{\sqrt{b}}+\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}\right)a}{4}$$

$$+\frac{\ln\left(\frac{2\,a+2\,b+2\,(\tanh(x)-1)\,b+2\sqrt{a+b}\,\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}}{\tanh(x)-1}\right)\sqrt{a+b}\;a}{2}$$

$$-\frac{\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}\;a}{2}$$

$$+\frac{\ln\left(\frac{2\,a+2\,b+2\,(\tanh(x)-1)\,b+2\sqrt{a+b}\,\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}}{\tanh(x)-1}\right)\sqrt{a+b}\;b}{2}$$

$$-\frac{b^{3/2}\ln\left(\frac{(\tanh(x)-1)\,b+b}{\sqrt{b}}+\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}\right)}{2}-\frac{\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}\;b}{2}$$

$$-\frac{\left(b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b\right)^{3/2}}{6}+\frac{b\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\;\tanh(x)}{4}$$

$$+\frac{3\sqrt{b}\,\ln\left(\frac{(1+\tanh(x))\,b-b}{\sqrt{b}}+\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\right)a}{4}$$

$$+\frac{\ln\left(\frac{2a+2b-2\,(1+\tanh(x))\,b+2\sqrt{a+b}\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}{1+\tanh(x)}\right)\sqrt{a+b}\;a}{2}$$

$$-\frac{\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\;a}{2}+\frac{\ln\left(\frac{(1+\tanh(x))\,b-b}{\sqrt{b}}+\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\right)b^{3/2}}{2}$$

$$+\frac{\ln\left(\frac{2a+2b-2\,(1+\tanh(x))\,b+2\sqrt{a+b}\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}{1+\tanh(x)}\right)\sqrt{a+b}\;b}{2}$$

$$-\frac{\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}\;b}{2}$$

Problem 59: Result more than twice size of optimal antiderivative.

$$\int\sqrt{1+\tanh(x)^2}\,dx$$

Optimal(type 3, 25 leaves, 5 steps):

$$-\mathrm{arcsinh}(\tanh(x))+\mathrm{arctanh}\left(\frac{\sqrt{2}\,\tanh(x)}{\sqrt{1+\tanh(x)^2}}\right)\sqrt{2}$$

Result(type 3, 96 leaves):

$$-\frac{\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{2}-\mathrm{arcsinh}(\tanh(x))+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{2}+\frac{\sqrt{(1+\tanh(x))^2-2\tanh(x)}}{2}$$

$$-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)}{2}$$

Problem 60: Result more than twice size of optimal antiderivative.

$$\int \left(-1-\tanh(x)^2\right)^{3/2}\, dx$$

Optimal(type 3, 53 leaves, 7 steps):

$$-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh(x)^2}}\right)}{2}+2\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh(x)^2}}\right)\sqrt{2}+\frac{\sqrt{-1-\tanh(x)^2}\,\tanh(x)}{2}$$

Result(type 3, 210 leaves):

$$-\frac{\left(-(\tanh(x)-1)^2-2\tanh(x)\right)^{3/2}}{6}+\frac{\tanh(x)\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}$$
$$+\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}-\sqrt{2}\arctan\left(\frac{(-2\tanh(x)-2)\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)+\frac{\left(-(1+\tanh(x))^2+2\tanh(x)\right)^{3/2}}{6}$$
$$+\frac{\tanh(x)\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{4}-\sqrt{-(1+\tanh(x))^2+2\tanh(x)}$$
$$+\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)^4}{\sqrt{a+b\tanh(x)^2}}\, dx$$

Optimal(type 3, 70 leaves, 7 steps):

$$\frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2b}$$

Result(type 3, 177 leaves):

$$-\frac{\ln\left(\tanh(x)\sqrt{b}+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}}-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2b}+\frac{a\ln\left(\tanh(x)\sqrt{b}+\sqrt{a+b\tanh(x)^2}\right)}{2b^{3/2}}$$
$$+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}$$

$$-\frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}$$

Problem 62: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)^3}{\sqrt{a+b\tanh(x)^2}}\,dx$$

Optimal(type 3, 39 leaves, 5 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(x)^2}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh(x)^2}}{b}$$

Result(type 3, 128 leaves):

$$-\frac{\sqrt{a+b\tanh(x)^2}}{b}+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\,dx$$

Optimal(type 3, 23 leaves, 4 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(x)^2}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Result(type 3, 113 leaves):

$$\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}$$

$$+\frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}$$

Problem 64: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)^5}{\left(a+b\tanh(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 62 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(x)^2}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a^2}{b^2(a+b)\sqrt{a+b\tanh(x)^2}}-\frac{\sqrt{a+b\tanh(x)^2}}{b^2}$$

Result(type 3, 321 leaves):

$$\frac{1}{b\sqrt{a+b\tanh(x)^2}}-\frac{\tanh(x)^2}{b\sqrt{a+b\tanh(x)^2}}-\frac{2a}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}$$
$$+\frac{b(2(\tanh(x)-1)b+2b)}{(a+b)\left(4b(a+b)-4b^2\right)\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2(a+b)^{3/2}}$$
$$-\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}-\frac{b(2(1+\tanh(x))b-2b)}{(a+b)\left(4b(a+b)-4b^2\right)\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2(a+b)^{3/2}}$$

Problem 65: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)^4}{\left(a+b\tanh(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 70 leaves, 7 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right)}{b^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right)}{(a+b)^{3/2}}+\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh(x)^2}}$$

Result(type 3, 327 leaves):

$$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{b\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\tanh(x)\sqrt{b}+\sqrt{a+b\tanh(x)^2}\right)}{b^{3/2}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}$$
$$+\frac{b(2(\tanh(x)-1)b+2b)}{(a+b)\left(4b(a+b)-4b^2\right)\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2(a+b)^{3/2}}$$
$$+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}+\frac{b(2(1+\tanh(x))b-2b)}{(a+b)\left(4b(a+b)-4b^2\right)\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}$$
$$-\frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2(a+b)^{3/2}}$$

Problem 66: Result more than twice size of optimal antiderivative.

$$\int\frac{\tanh(x)}{\left(a+b\tanh(x)^2\right)^{3/2}}\,dx$$

Optimal(type 3, 41 leaves, 5 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(x)^2}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{a+b\tanh(x)^2}}$$

Result(type 3, 272 leaves):

$$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}+\frac{b(2(\tanh(x)-1)b+2b)}{(a+b)\left(4b(a+b)-4b^2\right)\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2(a+b)^{3/2}}$$
$$-\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}-\frac{b(2(1+\tanh(x))b-2b)}{(a+b)\left(4b(a+b)-4b^2\right)\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}$$
$$+\frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2(a+b)^{3/2}}$$

Problem 67: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b\tanh(x)^2\right)^{3/2}}\, dx$$

Optimal(type 3, 48 leaves, 4 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a\,(a+b)\,\sqrt{a+b\tanh(x)^2}}$$

Result(type 3, 271 leaves):

$$-\frac{1}{2\,(a+b)\,\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}}+\frac{b\,(2\,(\tanh(x)-1)\,b+2\,b)}{(a+b)\,\left(4\,b\,(a+b)-4\,b^2\right)\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}}$$
$$+\frac{\ln\left(\frac{2\,a+2\,b+2\,(\tanh(x)-1)\,b+2\sqrt{a+b}\,\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}}{\tanh(x)-1}\right)}{2\,(a+b)^{3/2}}$$
$$+\frac{1}{2\,(a+b)\,\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}+\frac{b\,(2\,(1+\tanh(x))\,b-2\,b)}{(a+b)\,\left(4\,b\,(a+b)-4\,b^2\right)\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}$$
$$-\frac{\ln\left(\frac{2\,a+2\,b-2\,(1+\tanh(x))\,b+2\sqrt{a+b}\,\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}{1+\tanh(x)}\right)}{2\,(a+b)^{3/2}}$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int \frac{\tanh(x)^5}{\left(a+b\tanh(x)^2\right)^{5/2}}\, dx$$

Optimal(type 3, 72 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(x)^2}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{a\,(a+2\,b)}{b^2\,(a+b)^2\sqrt{a+b\tanh(x)^2}}-\frac{a^2}{3\,b^2\,(a+b)\,\left(a+b\tanh(x)^2\right)^{3/2}}$$

Result(type 3, 468 leaves):

$$\frac{1}{3\,b\,\left(a+b\tanh(x)^2\right)^{3/2}}+\frac{\tanh(x)^2}{b\,\left(a+b\tanh(x)^2\right)^{3/2}}+\frac{2\,a}{3\,b^2\,\left(a+b\tanh(x)^2\right)^{3/2}}-\frac{1}{6\,(a+b)\,\left(b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b\right)^{3/2}}$$
$$+\frac{b\tanh(x)}{6\,(a+b)\,a\,\left(b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b\right)^{3/2}}+\frac{b\tanh(x)}{3\,(a+b)\,a^2\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}}$$
$$-\frac{1}{2\,(a+b)^2\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}}+\frac{\tanh(x)\,b}{2\,(a+b)^2\,a\sqrt{b\,(\tanh(x)-1)^2+2\,(\tanh(x)-1)\,b+a+b}}$$

$$+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2(a+b)^{5/2}}$$

$$-\frac{1}{6(a+b)\left(b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b\right)^{3/2}}-\frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b\right)^{3/2}}$$

$$-\frac{b\tanh(x)}{3(a+b)a^2\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}-\frac{1}{2(a+b)^2\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}$$

$$-\frac{\tanh(x)\,b}{2(a+b)^2a\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}$$

$$+\frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2(a+b)^{5/2}}$$

Problem 69: Result more than twice size of optimal antiderivative.

$$\int\frac{\tanh(x)^2}{\left(a+b\tanh(x)^2\right)^{5/2}}\,dx$$

Optimal(type 3, 74 leaves, 6 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\,\tanh(x)}{\sqrt{a+b\tanh(x)^2}}\right)}{(a+b)^{5/2}}-\frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh(x)^2}}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh(x)^2\right)^{3/2}}$$

Result(type 3, 453 leaves):

$$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{3/2}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b\right)^{3/2}}$$

$$+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b\right)^{3/2}}+\frac{b\tanh(x)}{3(a+b)a^2\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}$$

$$-\frac{1}{2(a+b)^2\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}+\frac{\tanh(x)\,b}{2(a+b)^2a\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}$$

$$+\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2(a+b)^{5/2}}$$

$$+\frac{1}{6(a+b)\left(b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b\right)^{3/2}}+\frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^2-2(1+\tanh(x))b+a+b\right)^{3/2}}$$

$$+\frac{b\tanh(x)}{3\,(a+b)\,a^2\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}+\frac{1}{2\,(a+b)^2\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}$$

$$+\frac{\tanh(x)\,b}{2\,(a+b)^2\,a\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}$$

$$-\frac{\ln\left(\frac{2\,a+2\,b-2\,(1+\tanh(x))\,b+2\sqrt{a+b}\,\sqrt{b\,(1+\tanh(x))^2-2\,(1+\tanh(x))\,b+a+b}}{1+\tanh(x)}\right)}{2\,(a+b)^{5/2}}$$

Problem 71: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\tanh(x)\,\left(a+b\tanh(x)^4\right)^{3/2}\,dx$$

Optimal(type 3, 101 leaves, 9 steps):

$$\frac{(a+b)^{3/2}\operatorname{arctanh}\left(\frac{a+b\tanh(x)^2}{\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2}-\frac{(3\,a+2\,b)\operatorname{arctanh}\left(\frac{\sqrt{b}\,\tanh(x)^2}{\sqrt{a+b\tanh(x)^4}}\right)\sqrt{b}}{4}-\frac{\sqrt{a+b\tanh(x)^4}\,\left(2\,a+2\,b+b\tanh(x)^2\right)}{4}$$

$$-\frac{\left(a+b\tanh(x)^4\right)^{3/2}}{6}$$

Result(type 4, 619 leaves):

$$-\frac{3\ln\left(2\sqrt{b}\,\tanh(x)^2+2\sqrt{a+b\tanh(x)^4}\right)\sqrt{b}\,a}{4}+\frac{b\,a\operatorname{arctanh}\left(\frac{2\,b\tanh(x)^2+2\,a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{\sqrt{a+b}}$$

$$-\frac{I\left(\frac{7}{5}\,b\,a+b^2\right)\sqrt{a}\,\sqrt{1-\frac{I\sqrt{b}\,\tanh(x)^2}{\sqrt{a}}}\,\sqrt{1+\frac{I\sqrt{b}\,\tanh(x)^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(\tanh(x)\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}},I\right)-\operatorname{EllipticE}\left(\tanh(x)\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}},I\right)\right)}{2\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tanh(x)^4}\,\sqrt{b}}$$

$$-\frac{I\left(-\frac{7}{5}\,b\,a-b^2\right)\sqrt{a}\,\sqrt{1-\frac{I\sqrt{b}\,\tanh(x)^2}{\sqrt{a}}}\,\sqrt{1+\frac{I\sqrt{b}\,\tanh(x)^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(\tanh(x)\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}},I\right)-\operatorname{EllipticE}\left(\tanh(x)\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}},I\right)\right)}{2\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tanh(x)^4}\,\sqrt{b}}$$

$$- \frac{\left(-\frac{5}{3} b a - b^2\right) \sqrt{1 - \frac{I\sqrt{b}\,\tanh(x)^2}{\sqrt{a}}}\,\sqrt{1 + \frac{I\sqrt{b}\,\tanh(x)^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tanh(x)\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}}, I\right)}{2\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}}\,\sqrt{a + b\tanh(x)^4}}$$

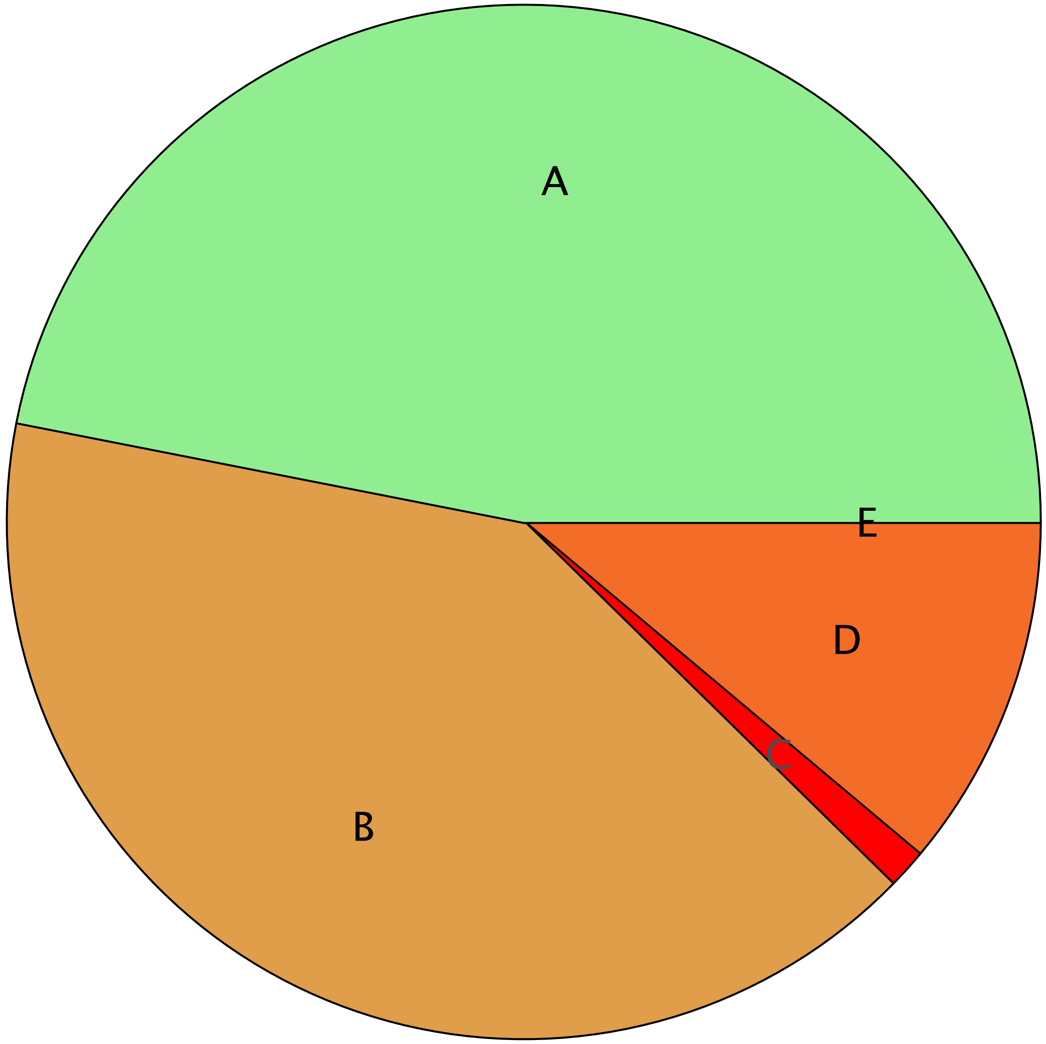
$$- \frac{\left(\frac{5}{3} b a + b^2\right) \sqrt{1 - \frac{I\sqrt{b}\,\tanh(x)^2}{\sqrt{a}}}\,\sqrt{1 + \frac{I\sqrt{b}\,\tanh(x)^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tanh(x)\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}}, I\right)}{2\sqrt{\frac{I\sqrt{b}}{\sqrt{a}}}\,\sqrt{a + b\tanh(x)^4}} - \frac{\ln\left(2\sqrt{b}\,\tanh(x)^2 + 2\sqrt{a + b\tanh(x)^4}\right) b^{3/2}}{2}$$

$$+ \frac{a^2 \operatorname{arctanh}\left(\frac{2 b\tanh(x)^2 + 2a}{2\sqrt{a+b}\,\sqrt{a + b\tanh(x)^4}}\right)}{2\sqrt{a+b}} + \frac{b^2 \operatorname{arctanh}\left(\frac{2 b\tanh(x)^2 + 2a}{2\sqrt{a+b}\,\sqrt{a + b\tanh(x)^4}}\right)}{2\sqrt{a+b}} - \frac{b\tanh(x)^4\sqrt{a + b\tanh(x)^4}}{6}$$

$$- \frac{b\tanh(x)^2\sqrt{a + b\tanh(x)^4}}{4} - \frac{2\sqrt{a + b\tanh(x)^4}\,a}{3} - \frac{b\sqrt{a + b\tanh(x)^4}}{2}$$

Summary of Integration Test Results

162 integration problems



A - 76 optimal antiderivatives
B - 66 more than twice size of optimal antiderivatives
C - 2 unnecessarily complex antiderivatives
D - 18 unable to integrate problems
E - 0 integration timeouts